

DASP3rd Chapter 3 - Exercises

1 Basics

Consider a simple sampling rate conversion system with a conversion rate of $\frac{4}{3}$. The system consists of two upsampling blocks, each by a factor of 2, and one downsampling block by a factor of 3.

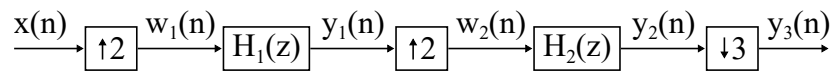
1. What are anti-imaging and anti-aliasing filters and where do we need them in our system?

Solution:

- Anti-imaging filters
 - * Upsampling creates unwanted spectral images
 - * Anti-imaging filters are used to remove these images
- Anti-aliasing filters
 - * Downsampling can move parts of the spectral images into the audible frequency range (aliasing)
 - * Anti-aliasing filters band-limit the signal to avoid this effect

2. Sketch the block diagram.

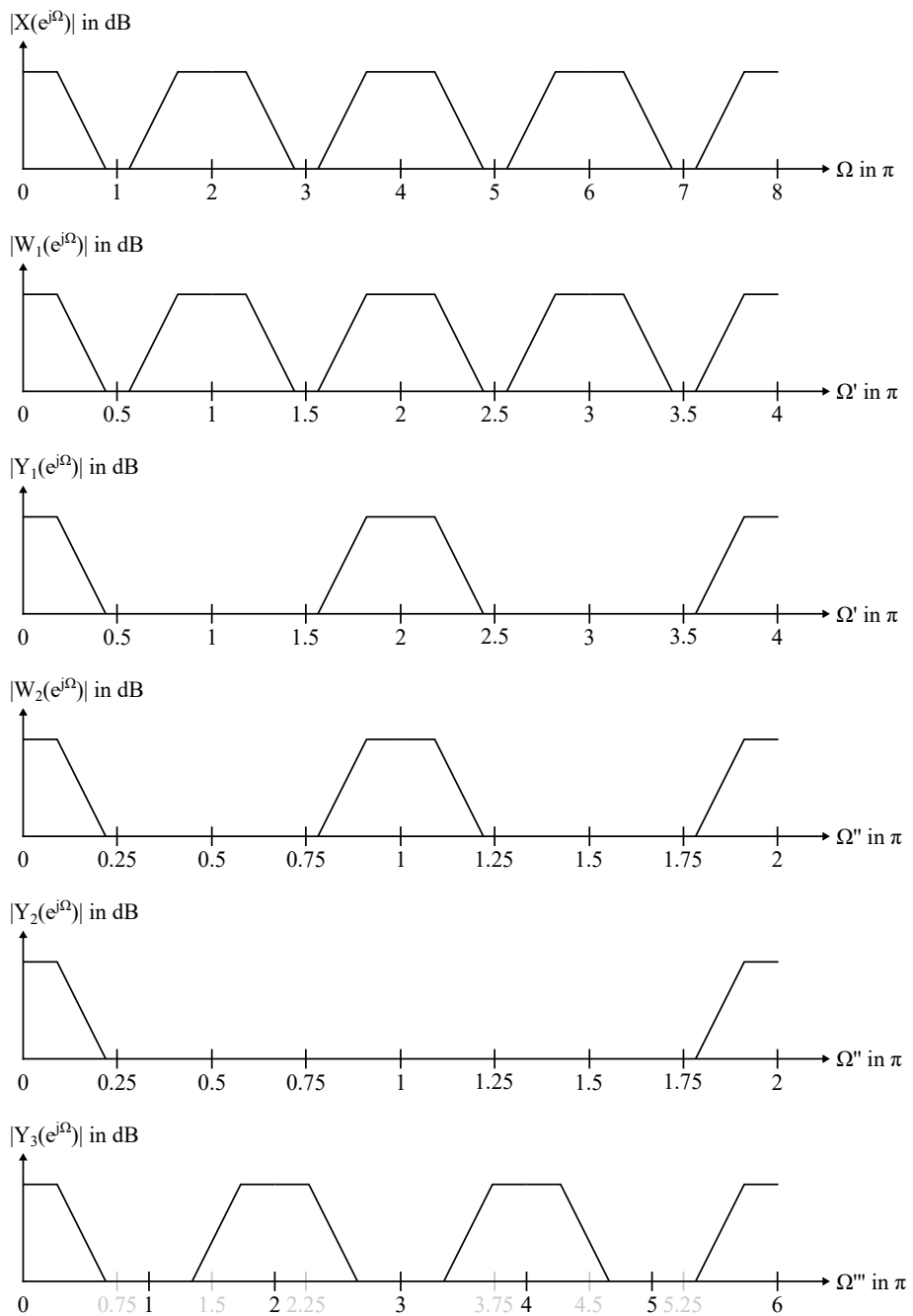
Solution:



- Where H_1 is an anti-imaging filter
- H_2 performs anti-imaging and anti-aliasing at the same time. The more restrictive filter defines the stop-band frequency (in this case the anti-aliasing operation).

3. Sketch the input, intermediate and output spectra in the frequency domain.

Solution:



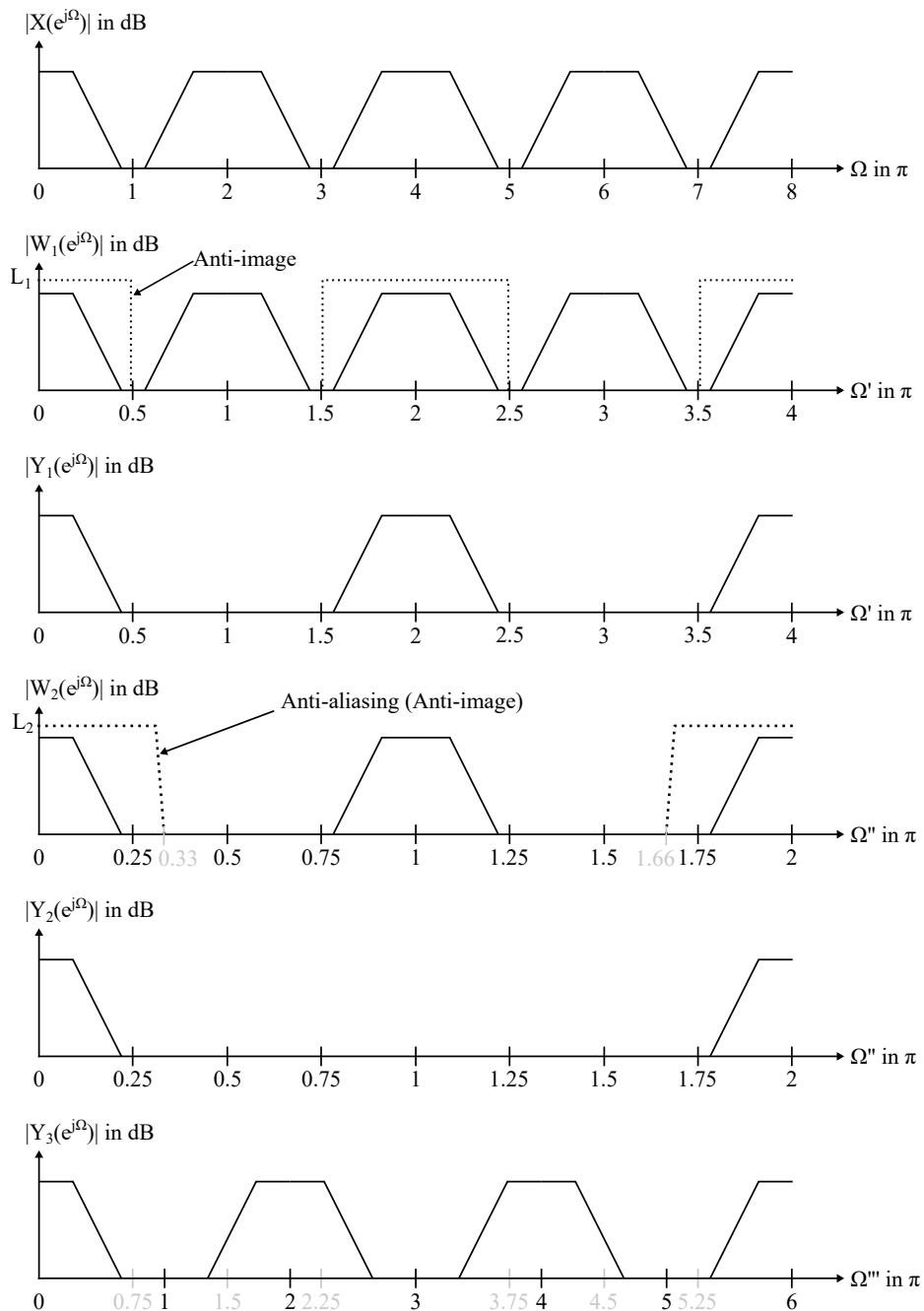
4. How is the amplitude affected by the up- and downsampling and where does it come from?

Solution:

- A gain factor L factor can be applied on the baseband signal after the upsampling operation, to keep the original amplitude. For the downsampling operation the amplitude is not affected if the anti-aliasing is done perfectly.

5. Sketch the frequency response of the anti-aliasing and anti-imaging filters needed for this up-sampling system.

Solution:



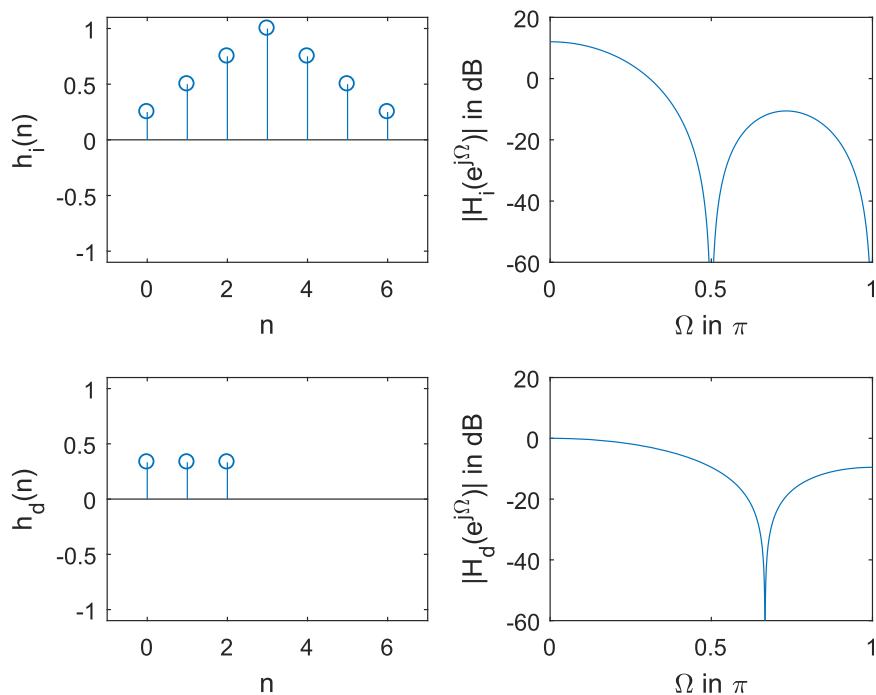
2 Synchronous Conversion

Our system will now be upsampled directly by a factor of 4 and again downsampled by the factor of 3, but with linear interpolation and decimation methods. The input signal is $x(n) = \sin(\frac{n\pi}{6})$, $n = 0, \dots, 48$.

1. What are the impulse responses of the two interpolation filters? Sketch their magnitude responses.

Solution:

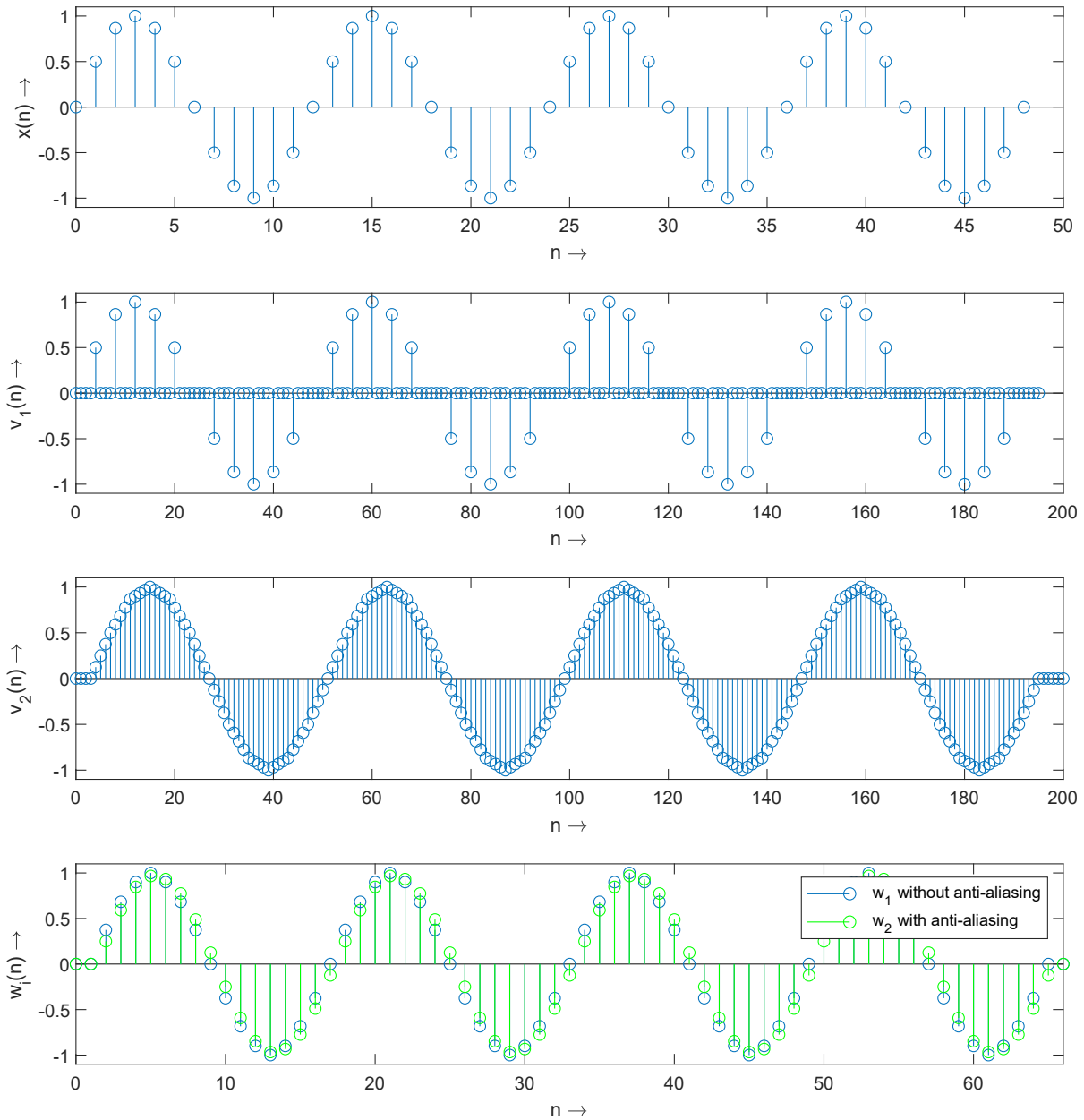
- see Matlab script 'dasp_ex3_2_1'



2. Plot the signals (input, intermediate, and output signal) in the time domain using Matlab.

Solution:

- see Matlab script 'dasp_ex3_2_2'



3. What is the delay resulting from the causal interpolation/decimation filters?

Solution:

$$d_i = \frac{N_i - 1}{2} = \frac{7 - 1}{2} = 3$$

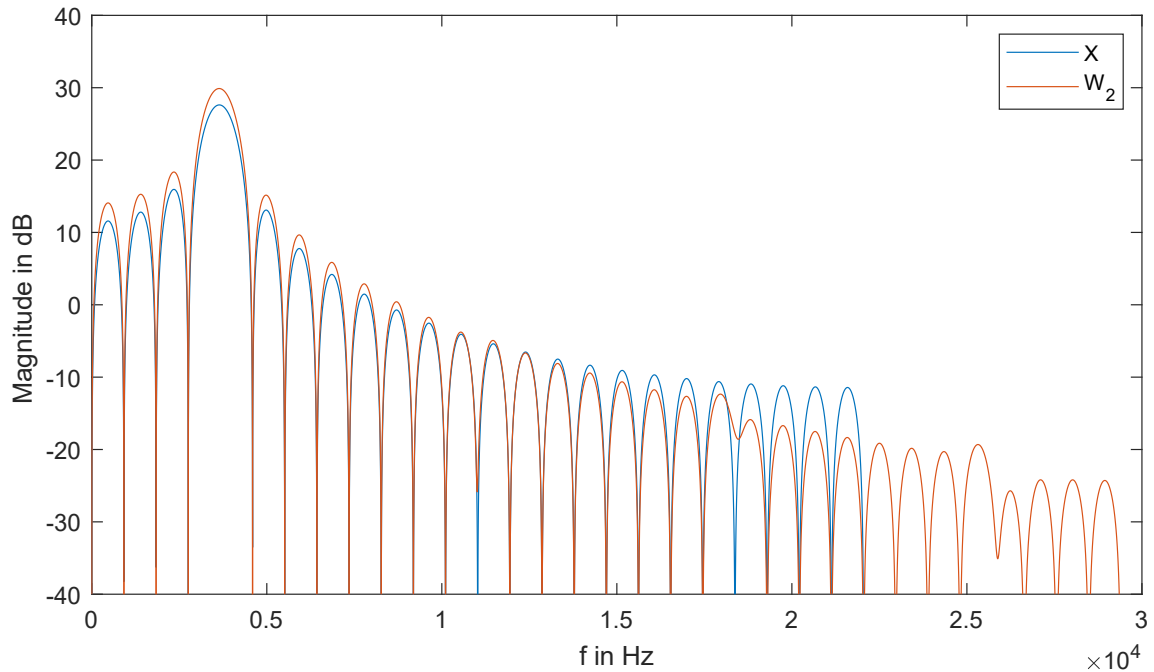
$$d_d = \frac{N_d - 1}{2} = \frac{3 - 1}{2} = 1$$

$$d_{tot} = \frac{d_i + d_d}{R_d} = \frac{3 + 1}{3} = \frac{4}{3}$$

4. Show the error introduced by this interpolation/decimation method in the frequency domain.

Solution:

- see Matlab script 'dasp_ex3_2_4'

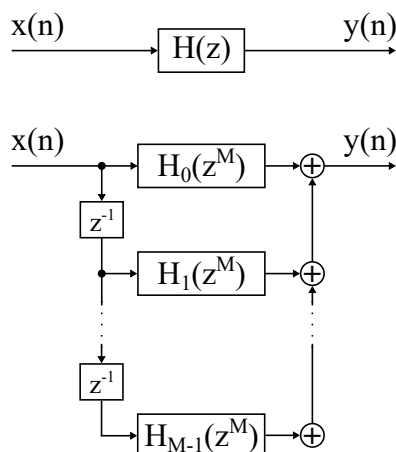


3 Polyphase Representation

Now we extend our system using a polyphase decomposition of the interpolation/decimation filters.

1. Sketch the idea of polyphase decomposition using a block diagram. What is the benefit of such decomposition?

Solution:



- Benefit: if N is the order of filter,
 - * Without polyphase decomposition: $N \times M$ multiplications/unit of time.
 - * With polyphase decomposition: $\frac{N}{M} \times M$ multiplications/unit of time.

2. Calculate the polyphase filters for up- and downsampling (using interpolation and decimation).

Solution:

- We are upsampling by a factor 4, so we have 4 filters. To build them we take every 4 sample of the original impulse response. For the next filter IR, we repeat the operation by shifting of 1 sample.

Interpolation filters:

$$h_{i1} = [0.25, 0.75]$$

$$h_{i2} = [0.50, 0.50]$$

$$h_{i3} = [0.75, 0.25]$$

$$h_{i4} = [1.00, 0.00]$$

We are downsampling by a factor 3, we repeat the operation on the decimation filter but taking every 3 sample.

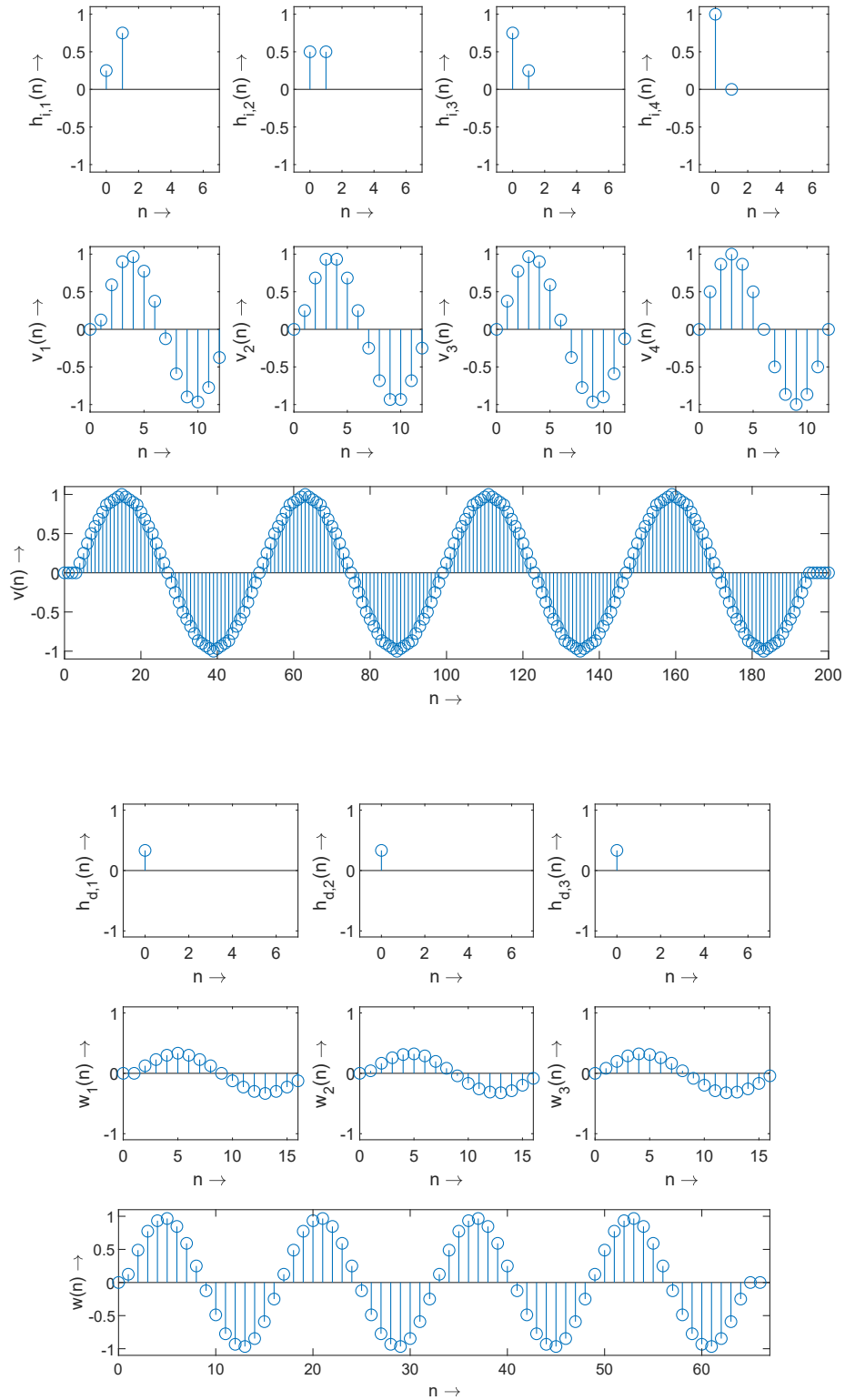
Decimation filters:

$$h_{d1} = \frac{1}{3}, h_{d2} = \frac{1}{3}, h_{d3} = \frac{1}{3}$$

3. Plot with Matlab all resulting signals in time and frequency domain.

Solution:

- see Matlab script 'dasp_ex3_3_3'



4 Asynchronous Conversion

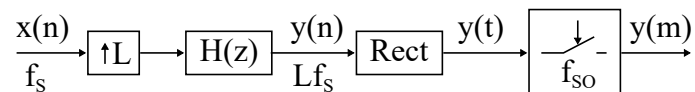
1. What is the basic concept of asynchronous sampling rate conversion?

Solution:

- The basic idea is to convert first the signal to the analog domain and then sampling it back at the desired sampling frequency.

2. Sketch the block diagram and discuss the individual operations.

Solution:



3. What is the necessary oversampling factor L for a 20-bit resolution?

Solution:

- for a wordlength $w = 20$, the oversampling factor L is defined by

$$L \geq 2^{w-1}$$

$$L \geq 2^{20-1}$$

$$L \geq 524288$$

4. How can we simplify the oversampling operations?

Solution:

- We can use a multistage oversampling system. Where the the first stages oversample by a factor 4 and the remaining oversampling is done by interpolating between the samples at a very high rate.
- We can use a linear interpolation instead of a sample-and-hold system. This can reduce the needed oversampling factor.

5. How can we make use of polyphase filtering?

Solution:

- We can use polyphase filtering to perform the first upsampling stages.

6. Why are half-band filters an efficient choice for the upsampling operation?

Solution:

- Because half-band filter needs to compute only half as many output samples as input samples. Wich reduces the number of operations.

7. Which parameters determine the interpolation algorithms in the last stage of the conversion?

Solution:

- The oversampling factors determine which algorithm to choose at the last stage of the conversion.