DASP3rd Chapter 2 - Exercises

1 Quantization

1. Consider a 100-Hz sine wave $x(n)$ sampled with $f_S = 44.1\,\text{kHz}$, $N = 1024$ number of samples, and $w = 3\,\text{bit}$ (word length). What is the number of quantization levels? What is the quantization step size Q when the signal is normalized to $-1 \leq x(n) < 1$. Show graphically how quantization is performed. What is the maximum error for this 3-bit quantizer? Write a Matlab code for quantization with rounding and truncation.

   Solution:

   - Number of quantization levels: $2^w = 2^3 = 8$
   - Quantization step size: $Q = \frac{2 \cdot x_{max}}{2^w}$ for $x_{max} = 1 \Rightarrow Q = 2^{-(w-1)} = 0.25$
   - A graphically explanation of quantization is shown in Matlab script 'dasp_ex2_1_1'
   - Maximum error: $e_{max} = \frac{Q}{2} = 0.125$ *(rounding)*
     $e_{max} = Q = 0.25$ *(truncation)*
   - Matlab: $x_r = Q \cdot \text{floor}\left(\frac{x}{Q} + 0.5\right)$ (rounding)
     $x_t = Q \cdot \text{floor}\left(\frac{x}{Q}\right)$ (truncation)

2. Derive the mean value, the variance, and the peak factor P_F of sequence $e(n)$ if the signal has a uniform probability density function (PDF) in the range $-\frac{Q}{2} < e(n) < \frac{Q}{2}$. Derive the signal-to-noise ratio (SNR) for this case. What will happen if we increase our word length by one bit?

   Solution:

   - Uniform PDF: $p_E(e) = \frac{1}{Q}\text{rect}\left(\frac{e}{Q}\right)$
   - Mean value: $m_E = E\{E\} = \int_{-\frac{Q}{2}}^{\frac{Q}{2}} e^1 \cdot p_E(e)de = \left[\frac{e^2}{2Q}\right]_{-\frac{Q}{2}}^{\frac{Q}{2}} = 0$
   - Variance: $\sigma_E^2 = E\{E^2\} = \int_{-\frac{Q}{2}}^{\frac{Q}{2}} e^2 \cdot p_E(e)de = \left[\frac{e^3}{3Q}\right]_{-\frac{Q}{2}}^{\frac{Q}{2}} = \frac{Q^2}{12}$
   - Peak factor: $P_F = \frac{x_{max}}{\sigma_E} \Rightarrow \sigma_E^2 = \frac{x_{max}^2}{P_F^2} \Rightarrow P_F = \sqrt{3}$
   - SNR $= 10\log_{10}\left(\frac{x_{max}^2/P_F^2}{\frac{1}{3}x_{max}^2 2^{-2w}}\right) = 10\log_{10}\left(2^{2w}\frac{3}{P_F^2}\right) = 6.02w - 10\log_{10}(P_F^2/3)\,\text{dB} = 6.02w\,\text{dB}$

3. As the input signal level decreases from maximum amplitude to very low amplitudes, the error signal becomes more audible. Describe the error calculated above when w decreases to 1 bit? Is the classical quantization model still valid? What can be done to avoid this distortion?

   Solution:

   - Error is now correlated with input and can be characterized as distortion of the signal since it is periodic.

- Due to the correlation of error and input, the classical quantization model is not valid.
- The nonlinear characteristic has to be linearized by adding dither.

4. Write a Matlab code for a quantizer with $w = 16$ bit with rounding and truncation.
   - Plot the nonlinear transfer characteristic and the error signal when the input signal covers the range $-3Q < x(n) < 3Q$.
   - Consider the sine wave $x(n) = Asin(2\pi \frac{f}{f_S} n)$, $n = 0, \ldots, N-1$ with $A = Q$, $\frac{f}{f_S} = 64/N$ and $N = 1024$. Plot the output signal (n = 0, . . . , 99) of a quantizer with rounding and truncation in the time domain and the frequency domain.
   - Compute for both quantization types the quantization error and the SNR.

   Solution:

   - see Matlab script 'dasp_ex2_1_4'

2 Dither

1. What is dither and when do we have to use it?

   Solution:

   - A random sequence in a range of $\pm\frac{Q}{2}$
   - During quantization of very low level signals

2. How do we perform dither and what kinds of dither do we have?

   Solution:

   - Add noise to the input signal with level -Q/2 < d(n) < Q/2
   - Rectangular dither, triangular dither, triangular highpass dither

3. How do we obtain a triangular highpass dither and why do we prefer it to other dithers?

   Solution:

   - $d_{triHP}(n) = d_1(n) - d_1(n-1)$ with uniform PDF random sequence $d_1(n)$
   - Preferred because of its shaping effect (see power density spectrum by noise shaping).

4. Matlab: Generate corresponding dither signals for rectangular, triangular and triangular highpass.

   Solution:

   - see Matlab script 'dasp_ex2_2_4'

5. Plot the amplitude distribution and the spectrum of the output $x_Q(n)$ of a quantizer for every dither type.

   Solution:

   - see Matlab script 'dasp_ex2_2_4'

3 Noise Shaping

1. What is noise shaping and when do we do it?

   Solution:

   - Modify the noise spectrum in order to move energy into frequency bands where humans have low sensitivity.

2. Why is it necessary to dither during noise shaping and how do we do this?

   Solution:

   - Make the quantization error uncorrelated to the input and cancel the shaped distortion effect.
   - Add dither to input prior to the feedback loop or in the feedback loop.

3. Matlab: The first noise shaper used is without dither and assumes that the transfer function in the feedback structure can be first-order $H(z) = z^{-1}$ or second-order $H(z) = 2z^{-1} - z^{-2}$. Plot the output $x_Q(n)$ and the error signal $e(n)$ and its spectrum. Show with a plot how the error signal will be shaped.

   Solution:

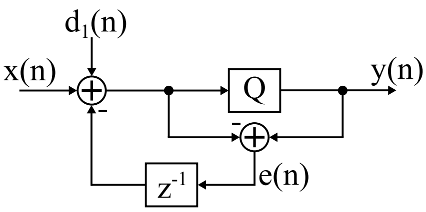
   - see Matlab script 'dasp_ex2_3_3'

4. The same noise shaper is now used with a dither signal. Is it really necessary to dither with noise shaping? Where would you add your dither in the flow graph to achieve better results?

   Solution:

   - Add dither to input prior to the feedback loop (i) or in the feedback loop (ii).
   - For better result we should add dither in the feedback loop prior the quantizer.

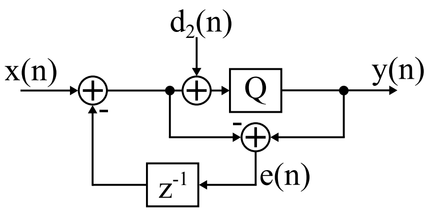
(i)



- dither and spectrum shaping of quantization error

$$y(n) = x(n) + d(n) + e(n) - e(n-1)$$

(ii)



- modified dither and spectrum shaping

$$y(n) = x(n) + d(n) - d(n-1) + e(n) - e(n-1)$$

5. In the feedback structure, we now use a psychoacoustic-based noise shaper which uses the Wannamaker filter coefficients:

$$h_3 = [1.623, -0.982, 0.109] \tag{1}$$
$$h_5 = [2.033, -2.165, 1.959, -1.590, 0.6149] \tag{2}$$
$$h_9 = [2.412, -3.370, 3.937, -4.174, 3.353, -2.205, 1.281, -0.569, 0.0847] \tag{3}$$

Show with a Matlab plot the shape of the error with this filter.

Solution:

- see Matlab script 'dasp_ex2_3_5'