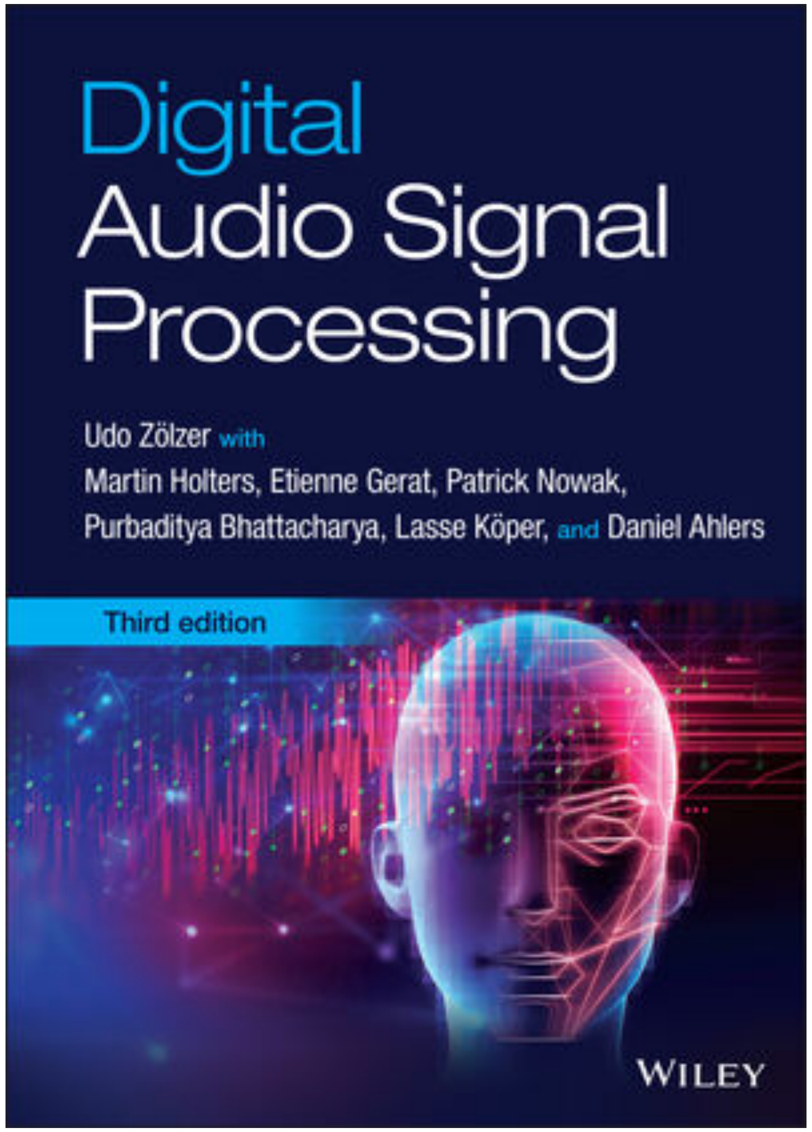


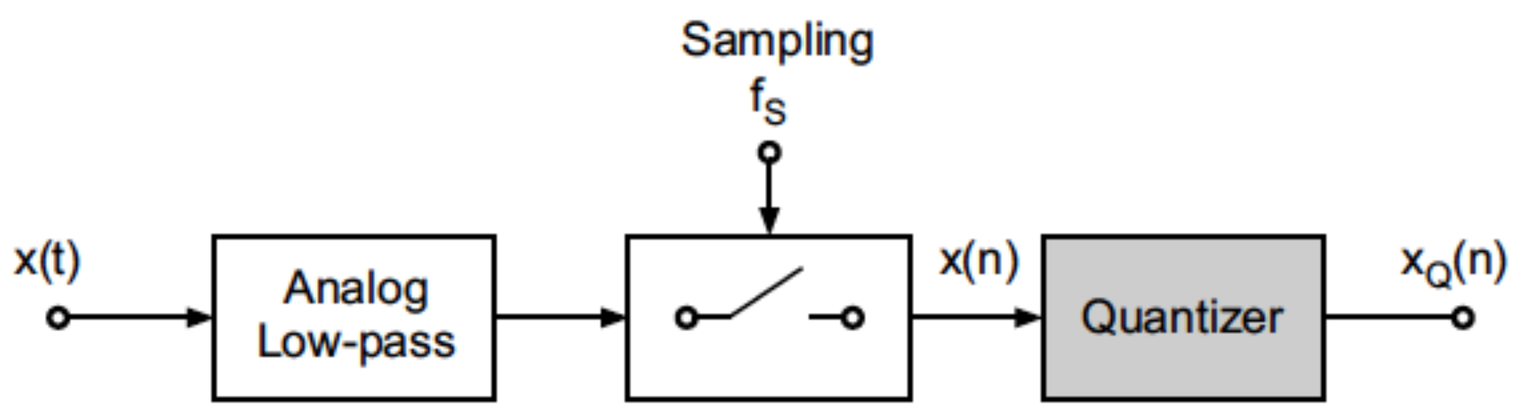
Lecture by Udo Zölzer

- Introduction
- Quantization
- Sampling Rate Conversion
- AD/DA Conversion
- Equalizers
- Room Simulation
- Dynamic Range Control
- Audio Coding
- Nonlinear Processing
- Machine Learning for Audio

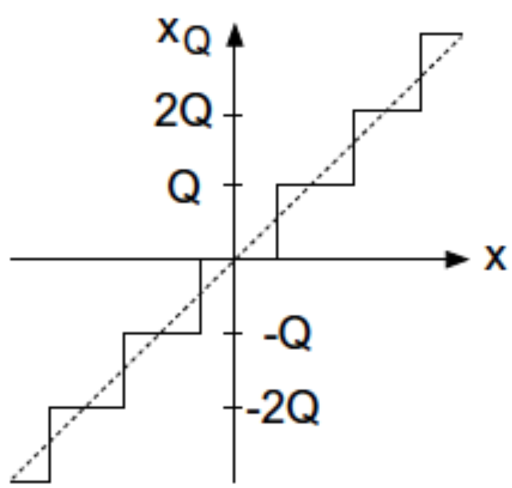
QUANTIZATION

OUTLINE

- Signal Quantization
- Dither
- Noise Shaping

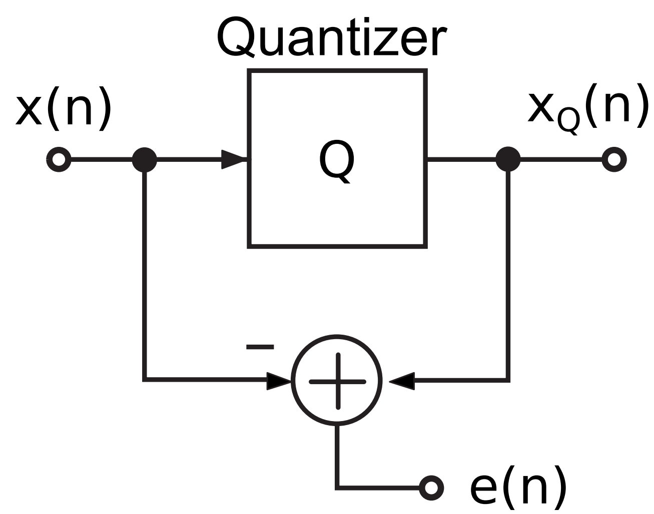


QUANTIZER – SIGNAL QUANTIZATION



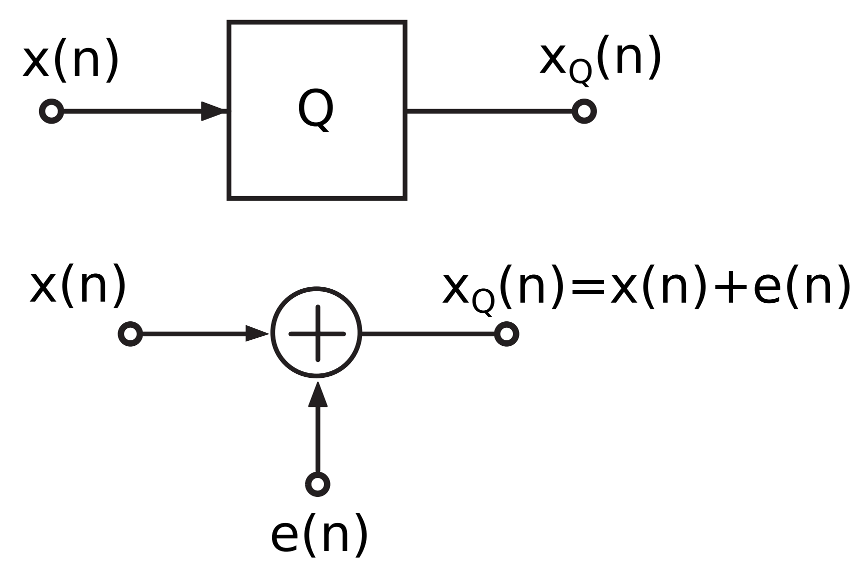
Nonlinear static curve $x_Q=f(x)$

Quantization step size Q



Quantization error

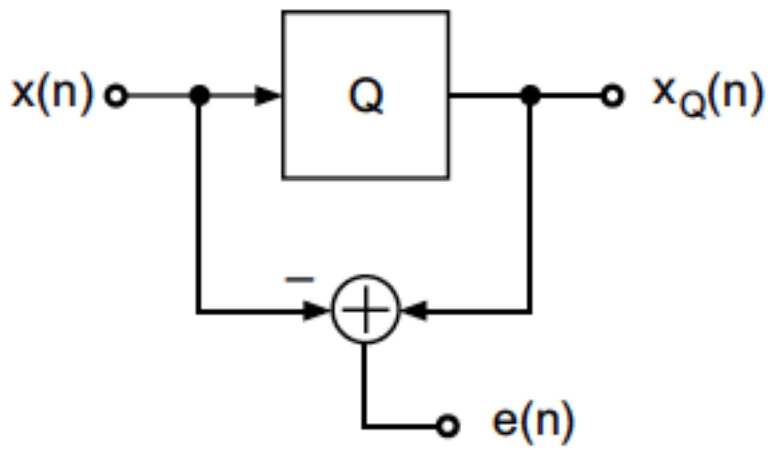
$$e(n) = x_Q(n) - x(n)$$

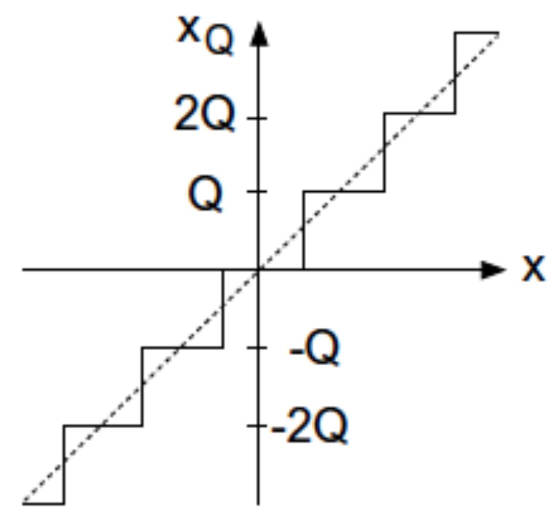


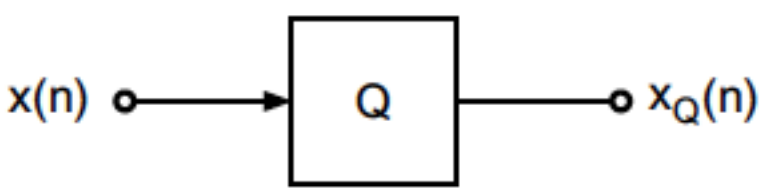
Linear model of a quantizer

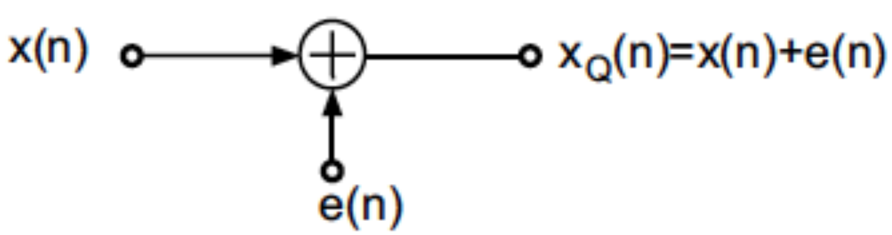
- addition of a random signal e(n) which has uniform probability density function (PDF)

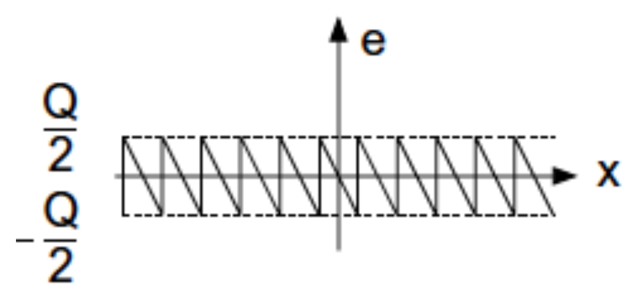
SIGNAL QUANTIZATION

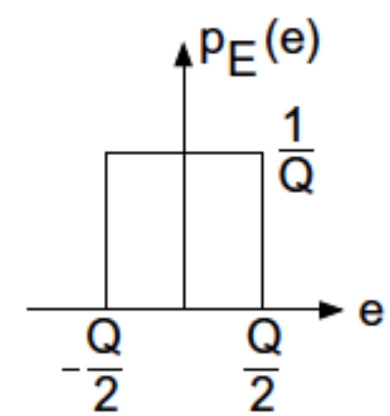












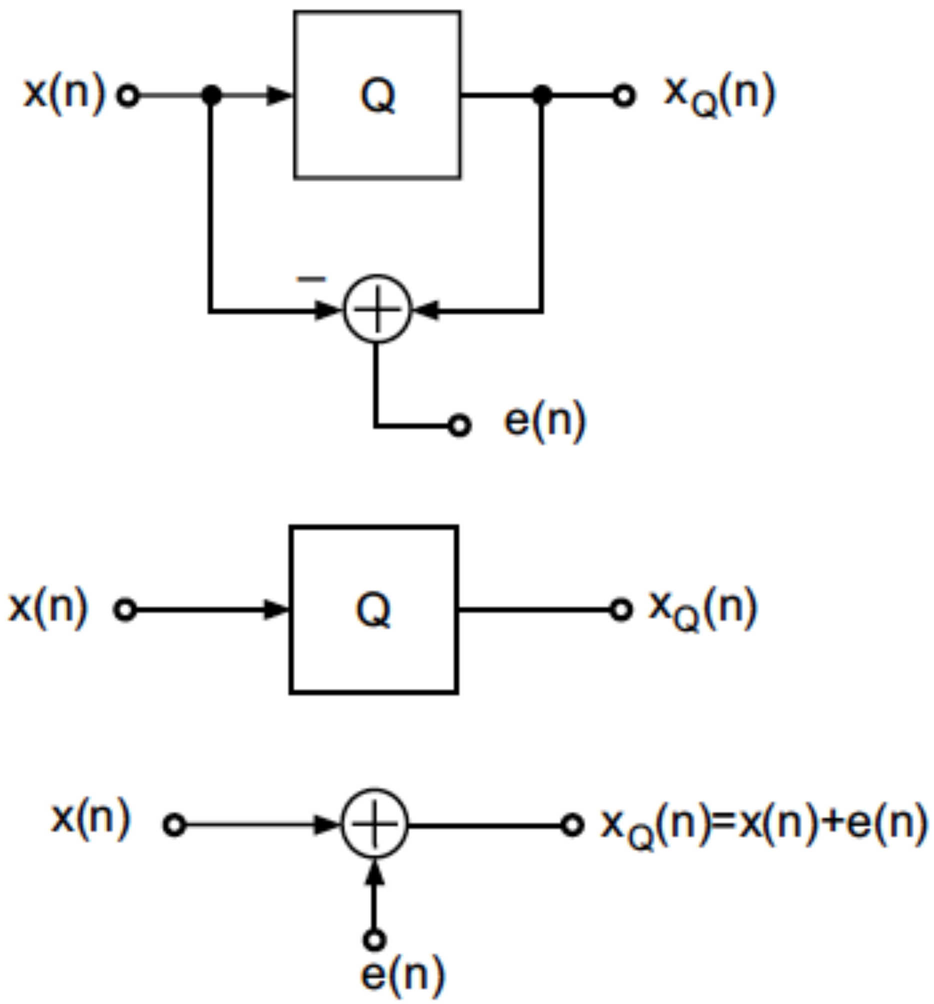
PDF $$p_E(e) = \frac{1}{Q}\mathrm{rect}\left(\frac{e}{Q}\right)$$

$$E[\mathbf{E}^m] = \int_{-\infty}^{\infty} e^m p_E(e)\, de.$$

$$m_E = E[\mathbf{E}] \quad = \quad 0 \qquad \text{mean value}$$

$$\sigma_E^2 = E[\mathbf{E}^2] \quad = \quad \frac{Q^2}{12} \qquad \text{variance.}$$

SIGNAL QUANTIZATION



Autocorrelation sequence of e(n)

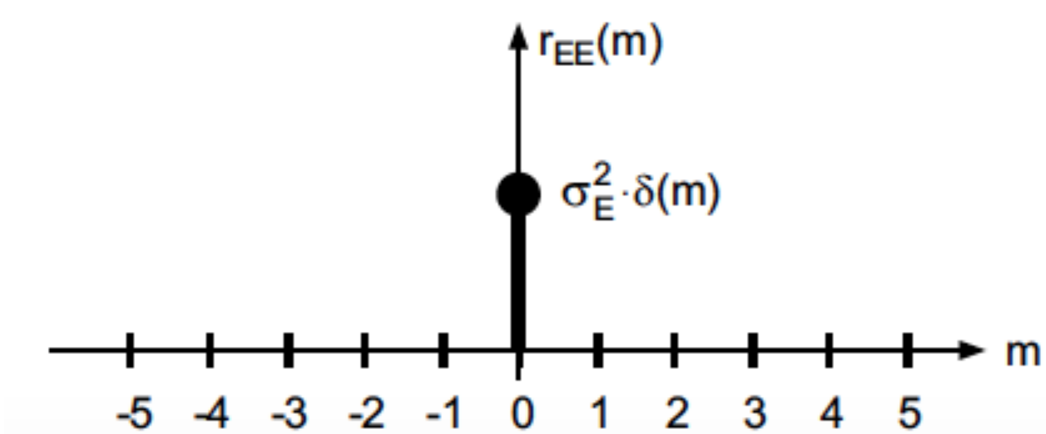
$$r_{EE}(m) = \sum_n e(n) \cdot e(n+m) = \frac{Q^2}{12}\delta(m)$$

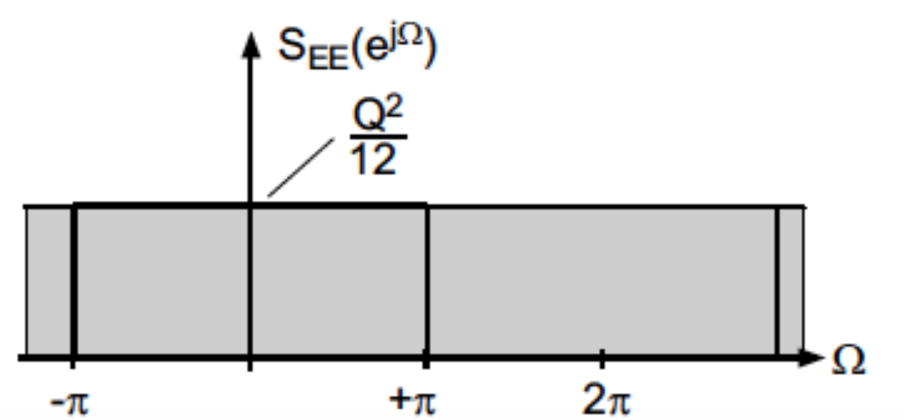
Power Density Spectrum of Quantization Error

$$S_{EE}(e^{j\Omega}) = \sum_m r_{EE}(m) e^{-j\Omega m} = \frac{Q^2}{12}$$

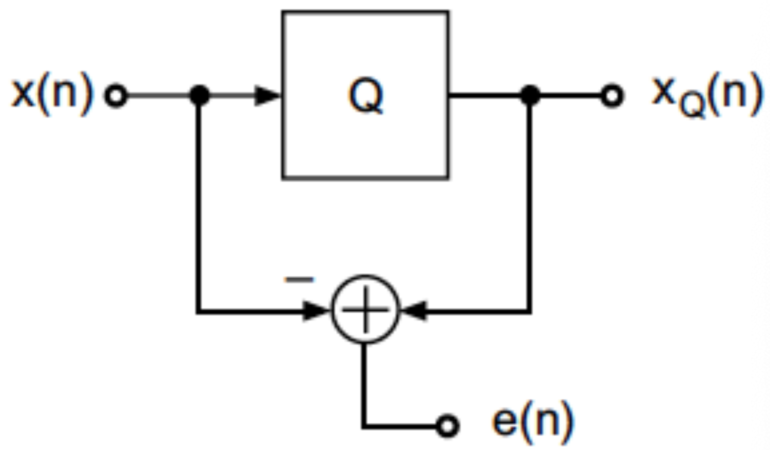
Power Density Spectrum of output sequence $x_Q(n)$

$$S_{X_Q X_Q}(e^{j\Omega}) = S_{XX}(e^{j\Omega}) + S_{EE}(e^{j\Omega})$$

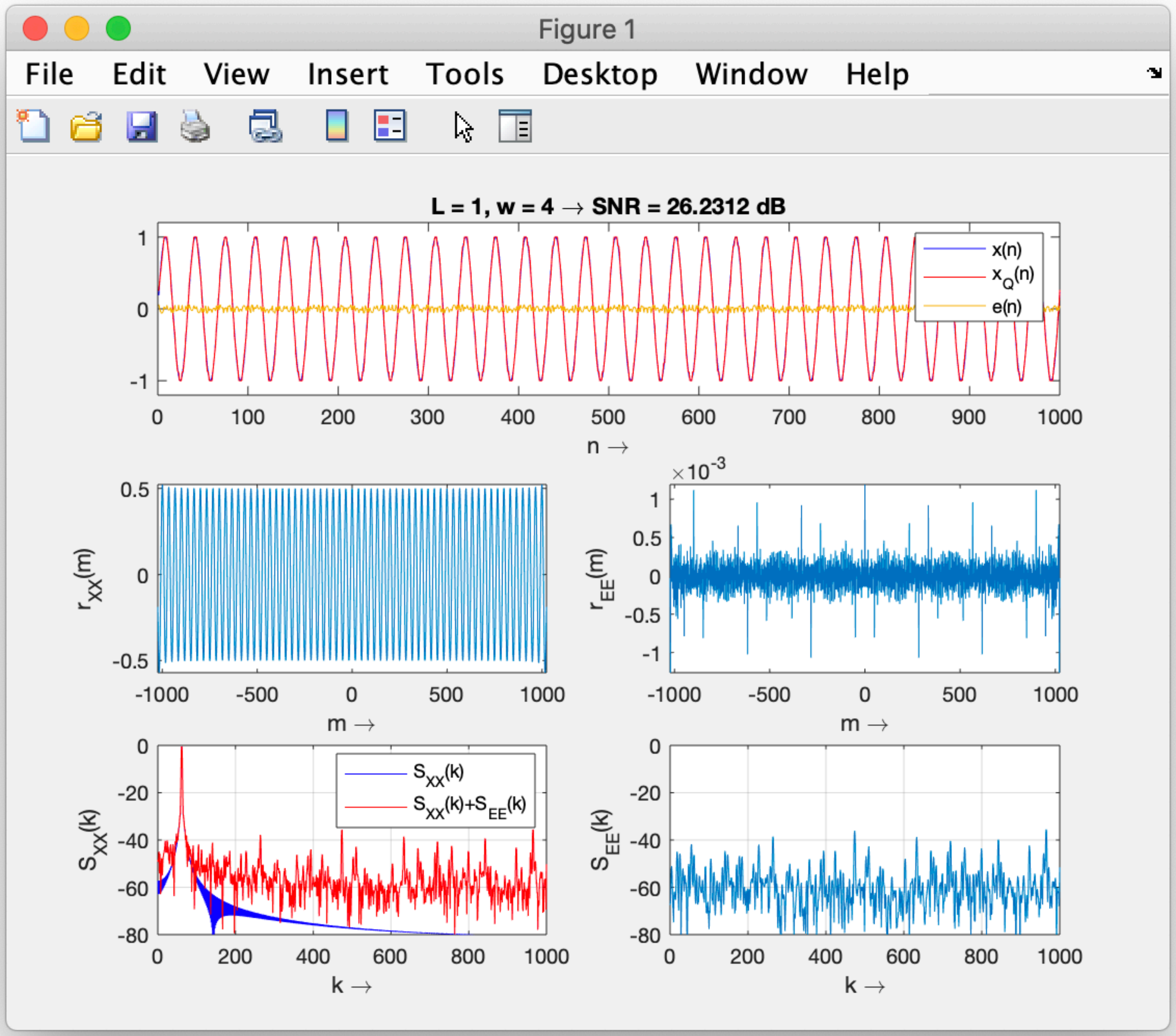




SIGNAL QUANTIZATION



os_quant.m



SIGNAL-TO-NOISE RATIO OF QUANTIZER

Quantizer stepsize Q
Quantizer word length w

$$Q = \frac{2x_{max}}{2^w} \qquad x_{\max} = 1 \rightarrow Q = 2^{-(w-1)}$$

Peak factor (crest factor)

$$P_F = \frac{x_{max}}{\sigma_X} = \frac{2^{w-1}Q}{\sigma_X}$$

Input variance (power)

$$\sigma_X^2 = \frac{x_{max}^2}{P_F^2} \quad \text{and}$$

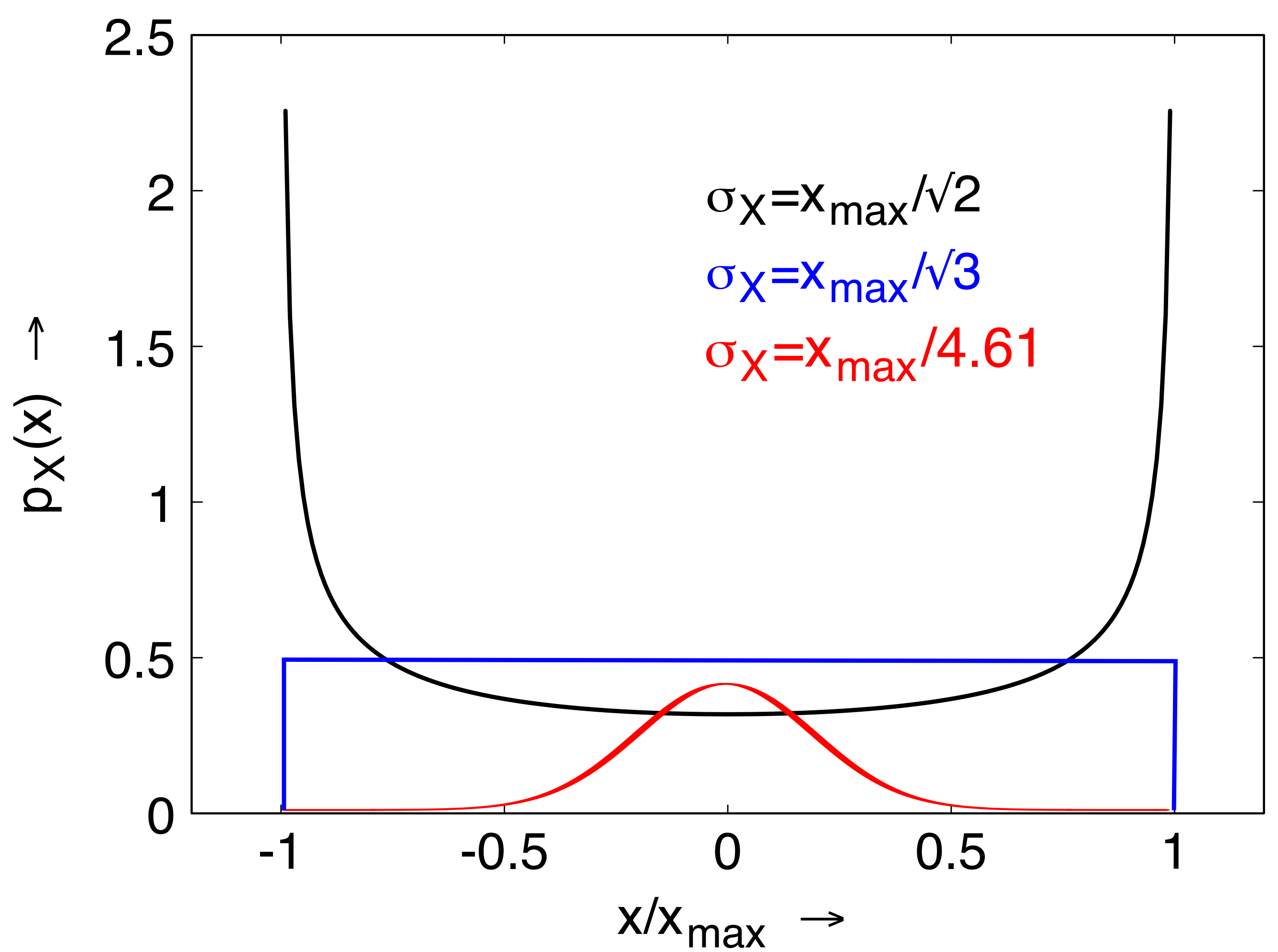
Error variance (power)

$$\sigma_E^2 = \frac{Q^2}{12} = \frac{1}{12}\frac{x_{max}^2}{2^{2w}}2^2 = \frac{1}{3}x_{max}^2 2^{-2w} \quad .$$

Signal-to-noise ratio

$$\begin{aligned} \text{SNR} &= 10\log_{10}\left(\frac{\sigma_X^2}{\sigma_E^2}\right) \\ &= 10\log_{10}\left(\frac{x_{max}^2/P_F^2}{\frac{1}{3}x_{max}^2 2^{-2w}}\right) = 10\log_{10}\left(2^{2w}\frac{3}{P_F^2}\right) \\ &= 6.02\,w - 10\log_{10}(P_F^2/3) \qquad \text{in dB} \end{aligned}$$

SNR FOR DIFFERENT INPUT PDF



	P_F	SNR in dB	
Sinus	$\sqrt{2}$	$6 \cdot w + 1.76$	
Uniform	$\sqrt{3}$	$6 \cdot w$	
Gauss	4.61	$6 \cdot w - 8.5$	probability of overload 10^{-5}

DITHER

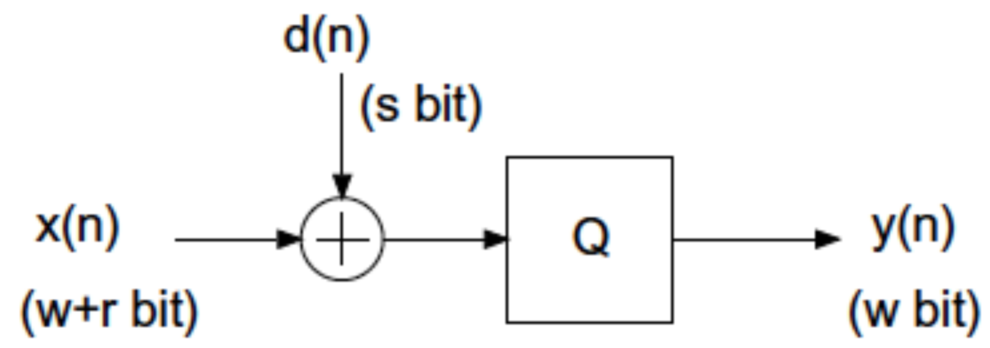


Figure 2.17 Addition of a random sequence before a quantizer.

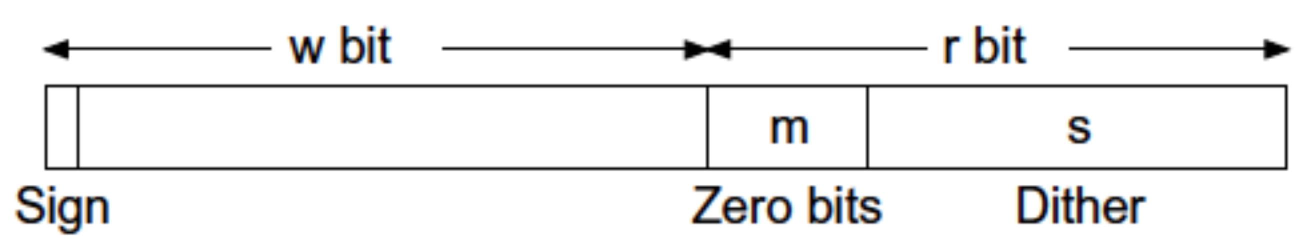
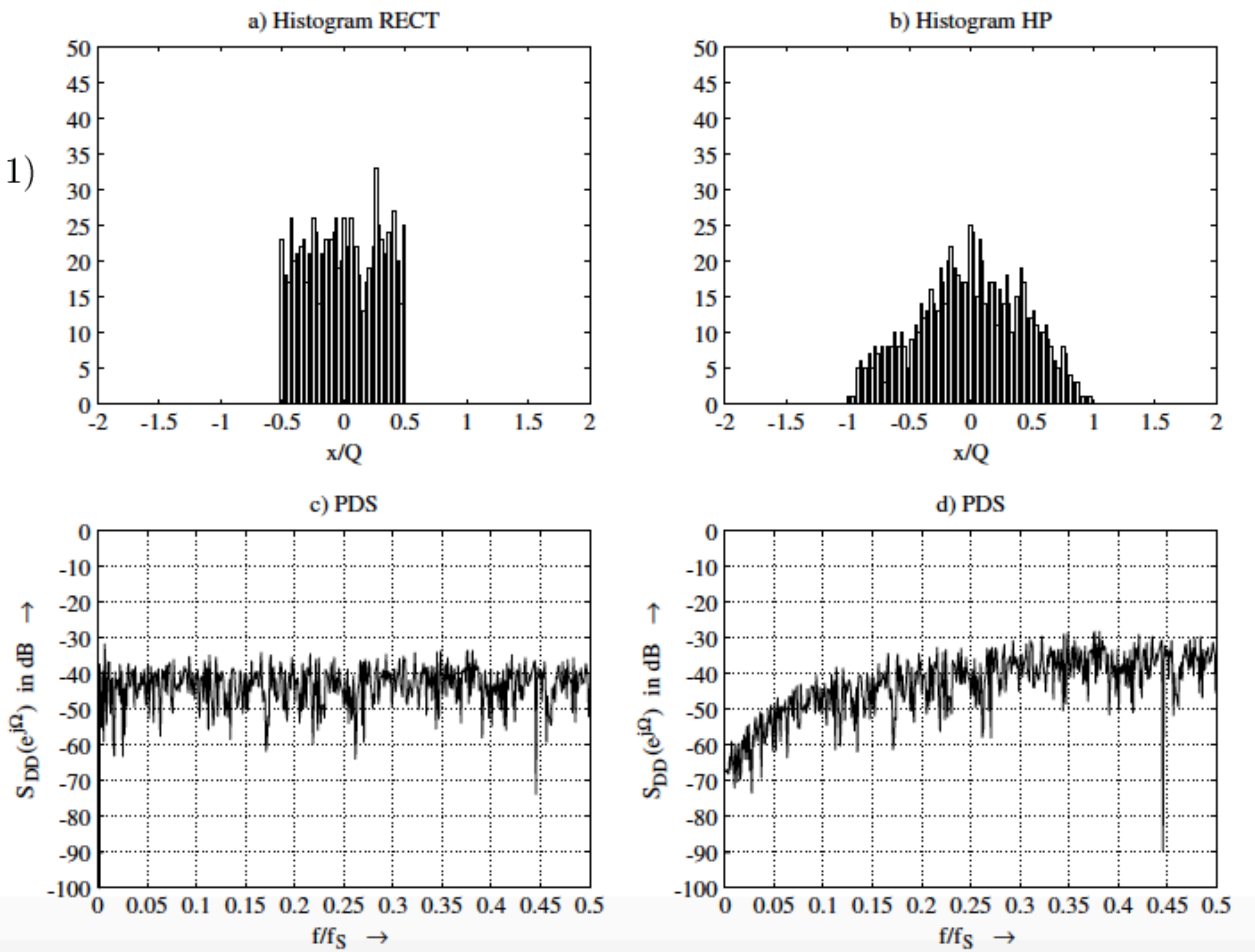


Figure 2.18 Specification of the word-length.

DITHER – HISTOGRAMS AND POWER DENSITY SPECTRA

$$
\begin{aligned}
d_{\mathrm{RECT}}(n) &= d_1(n) \\
d_{\mathrm{TRI}}(n) &= d_1(n) + d_2(n) \\
d_{\mathrm{HP}}(n) &= d_1(n) - d_1(n-1)
\end{aligned}
$$



DITHER SIGNALS + POWER DENSITY SPECTRA

$$\begin{aligned} d_{\mathrm{RECT}}(n) &= d_1(n) \\ d_{\mathrm{TRI}}(n) &= d_1(n) + d_2(n) \\ d_{\mathrm{HP}}(n) &= d_1(n) - d_1(n-1) \end{aligned}$$

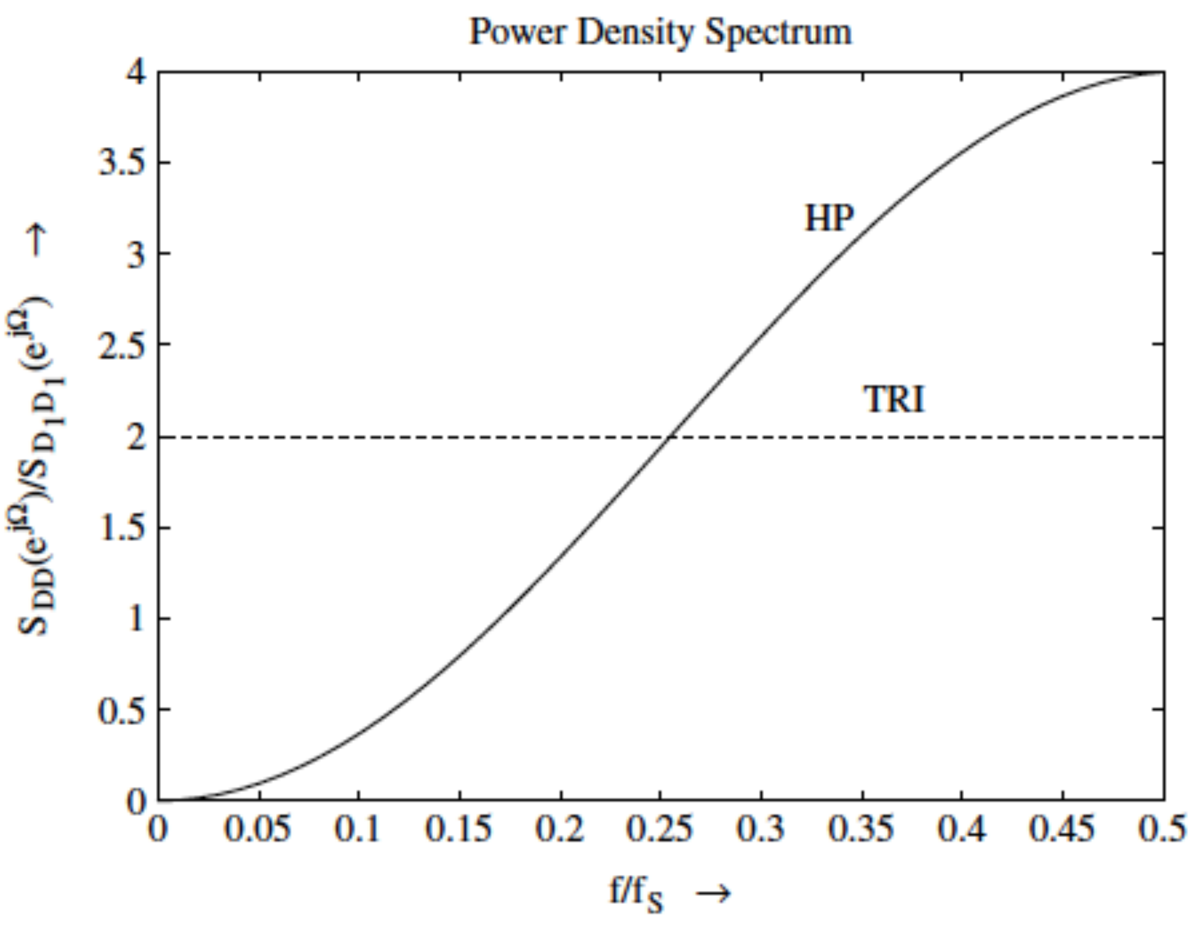
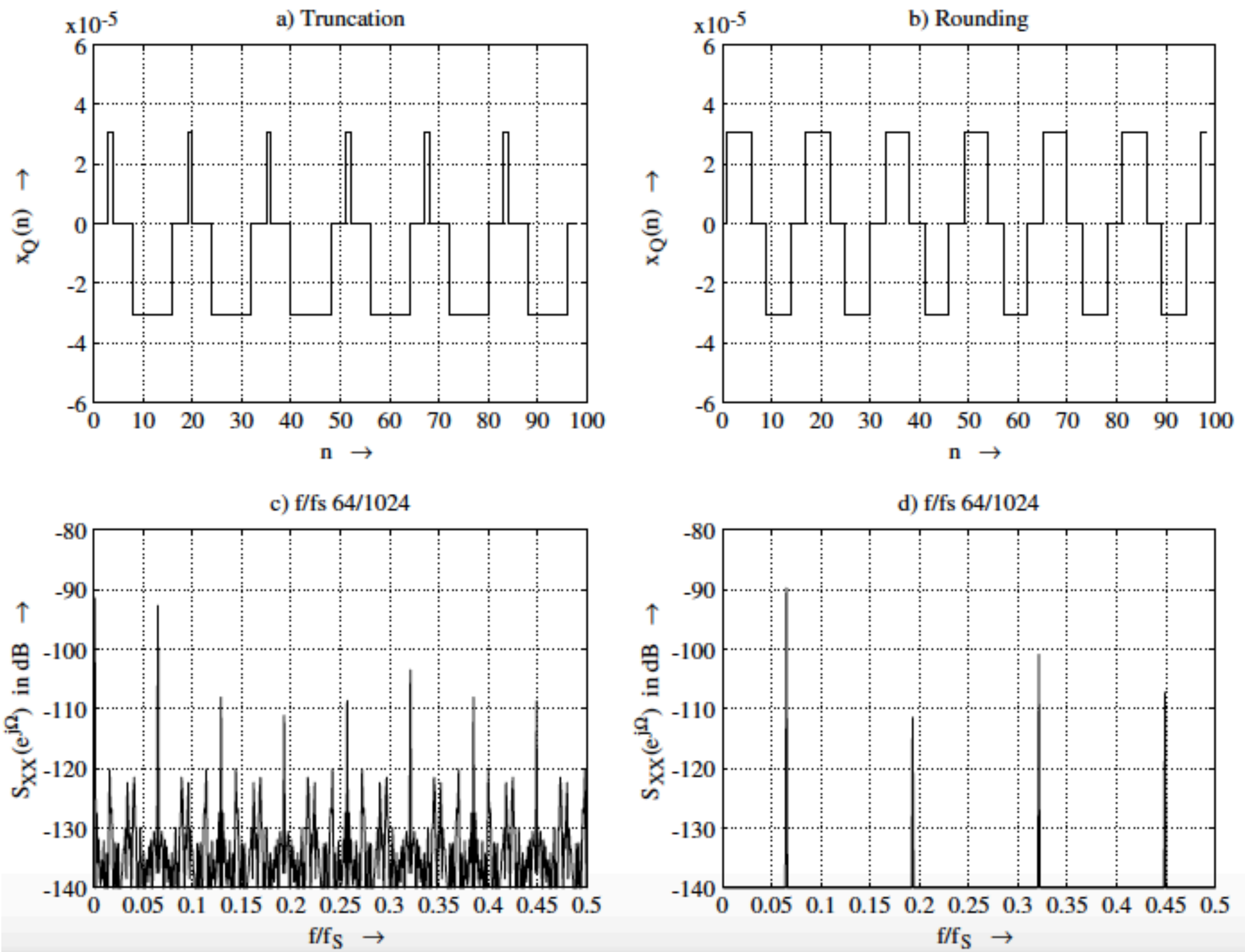
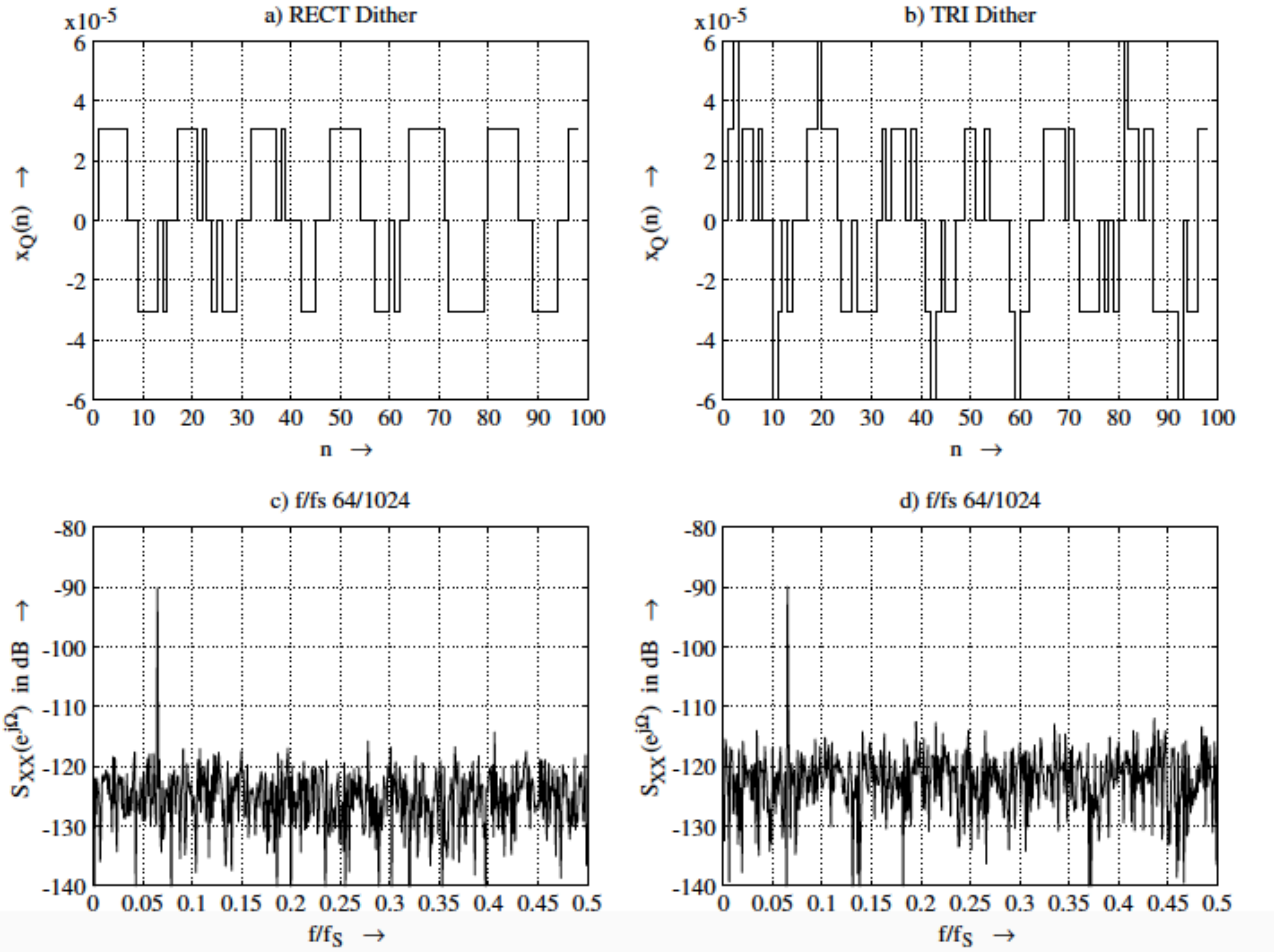


Figure 2.21 Normalized power density spectrum for triangular PDF dither (TRI) with $d_1(n) + d_2(n)$ and triangular PDF high-pass dither (HP) with $d_1(n) - d_1(n-1)$.

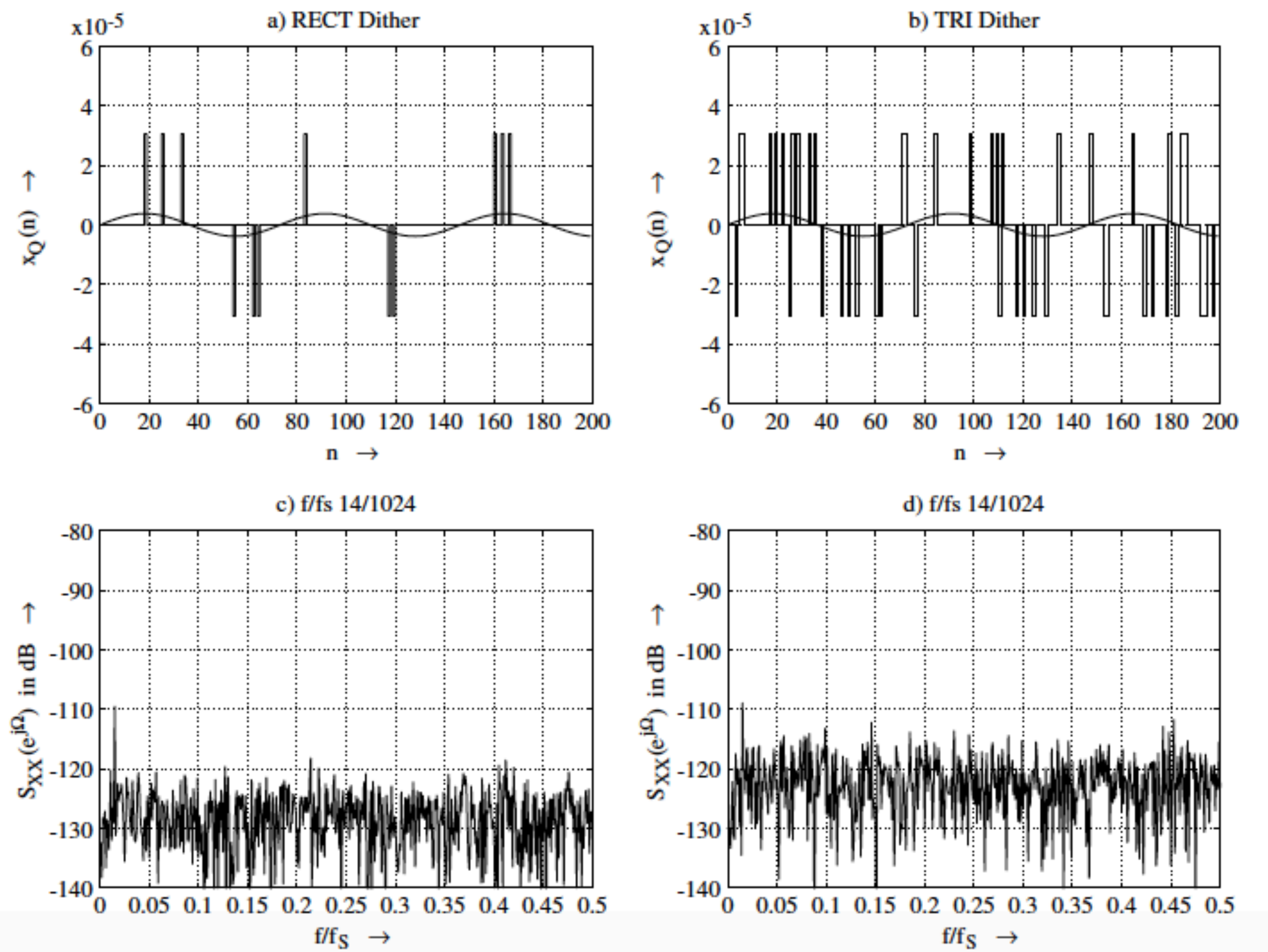
DITHER SIGNALS + POWER DENSITY SPECTRA



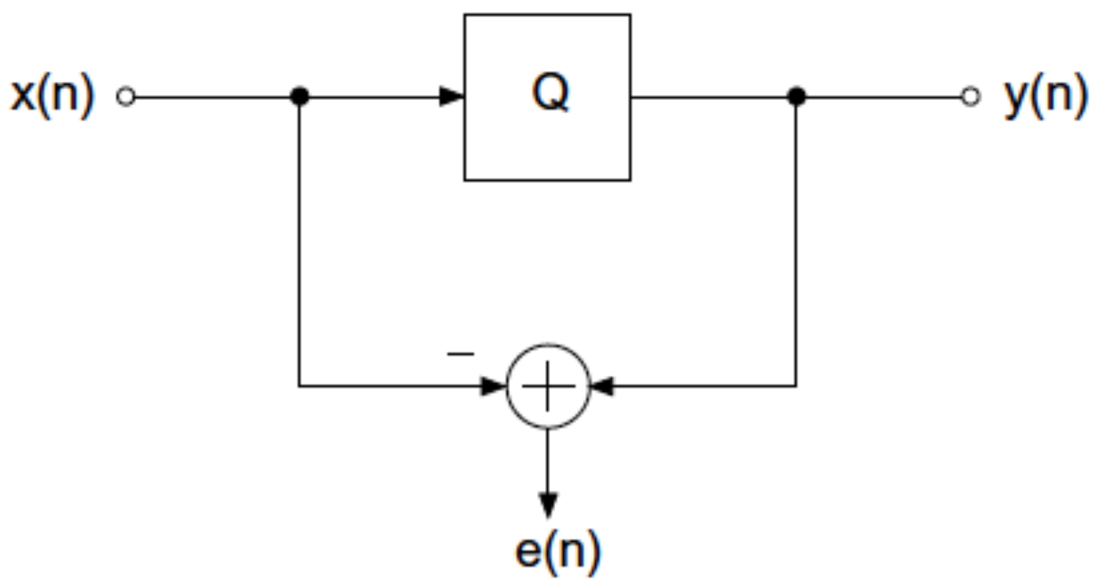
DITHER SIGNALS + POWER DENSITY SPECTRA

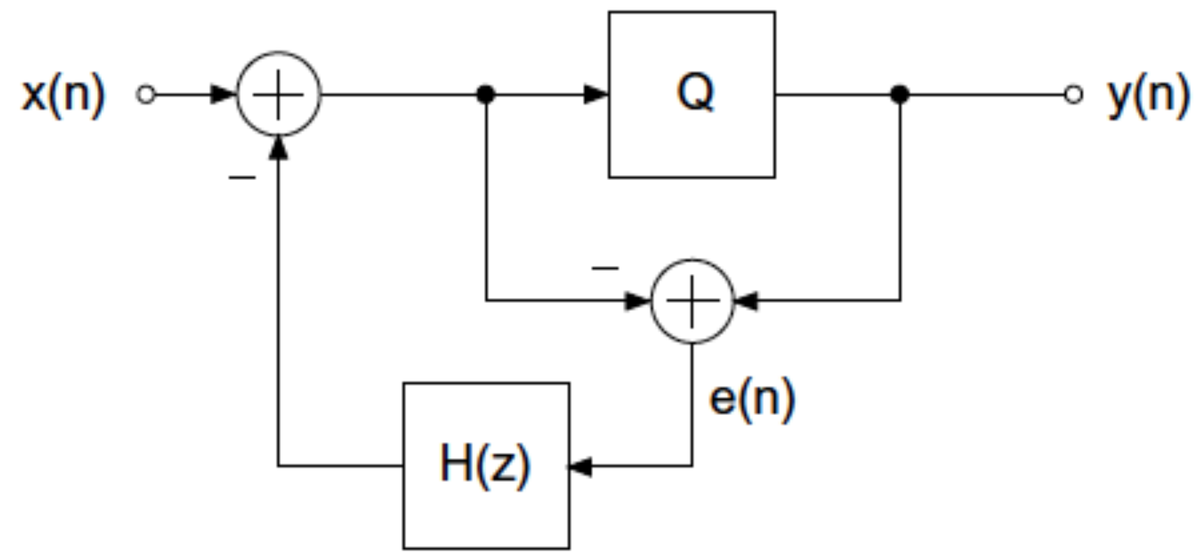


DITHER SIGNALS + POWER DENSITY SPECTRA



NOISE SHAPING – ERROR FEEDBACK



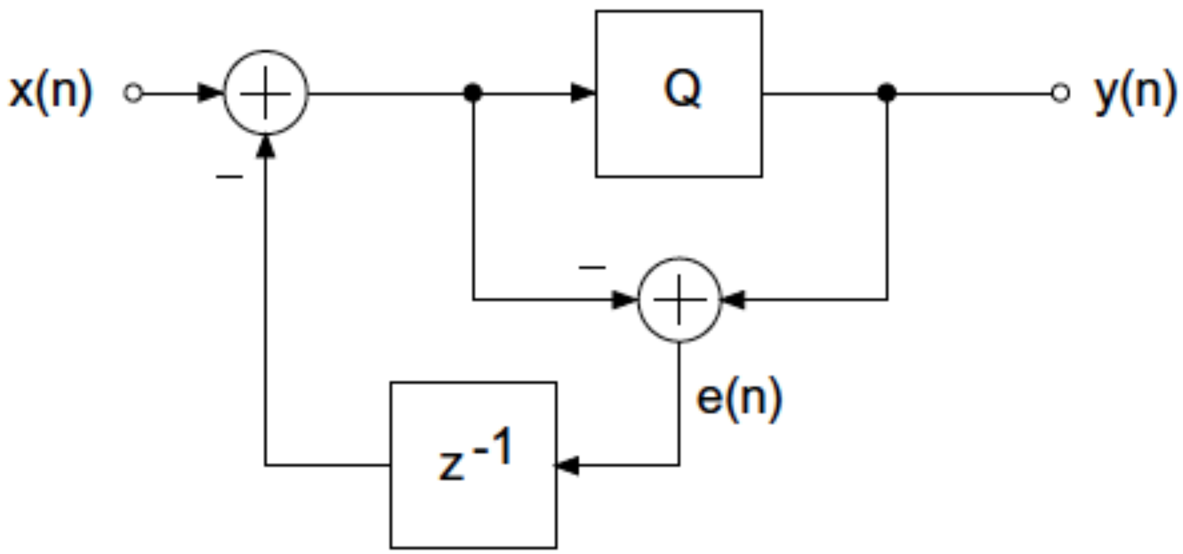


$$
\begin{aligned}
e(n) &= y(n) - x(n) \\
y(n) &= [x(n)]_Q \\
&= x(n) + e(n)
\end{aligned}
$$

$$
\begin{aligned}
y(n) &= [x(n) - e(n) \star h(n)]_Q \\
&= x(n) + e(n) - e(n) \star h(n) \\
e_1(n) &= y(n) - x(n) \\
&= e(n) \star [\delta(n) - h(n)]
\end{aligned}
$$

$$
\begin{aligned}
Y(z) &= X(z) + E(z)[1 - H(z)] \\
\Longrightarrow \quad E_1(z) &= E(z)[1 - H(z)] \quad .
\end{aligned}
$$

NOISE SHAPING – FIRST/SECOND ORDER FIR FILTERS



$$\begin{aligned} y(n) &= [x(n) - e(n-1)]_Q \\ &= x(n) - e(n-1) + e(n) \\ e_1(n) &= y(n) - x(n) \\ &= e(n) - e(n-1) \end{aligned}$$

$$\begin{aligned} Y(z) &= X(z) + E(z)[1 - z^{-1}] \\ E_1(z) &= E(z)[1 - z^{-1}]. \end{aligned}$$

$H_1(z) = z^{-1}$ $\Longrightarrow$ $E_1(z) = E(z)[1 - z^{-1}]$

$H_2(z) = 2z^{-1} - z^{-2}$ $\Longrightarrow$ $E_2(z) = E(z)[1 - 2z^{-1} + z^{-2}]$

Power Density Spectrum

$$\begin{aligned} S_{E_1 E_2}(e^{j\Omega}) &= |1 - e^{-j\Omega}|^2 \cdot S_{EE}(e^{j\Omega}) \\ S_{E_2 E_2}(e^{j\Omega}) &= |1 - 2e^{-j\Omega} + e^{-j2\Omega}|^2 \cdot S_{EE}(e^{j\Omega}) \end{aligned}$$

NOISE SHAPING IN FREQUENCY DOMAIN

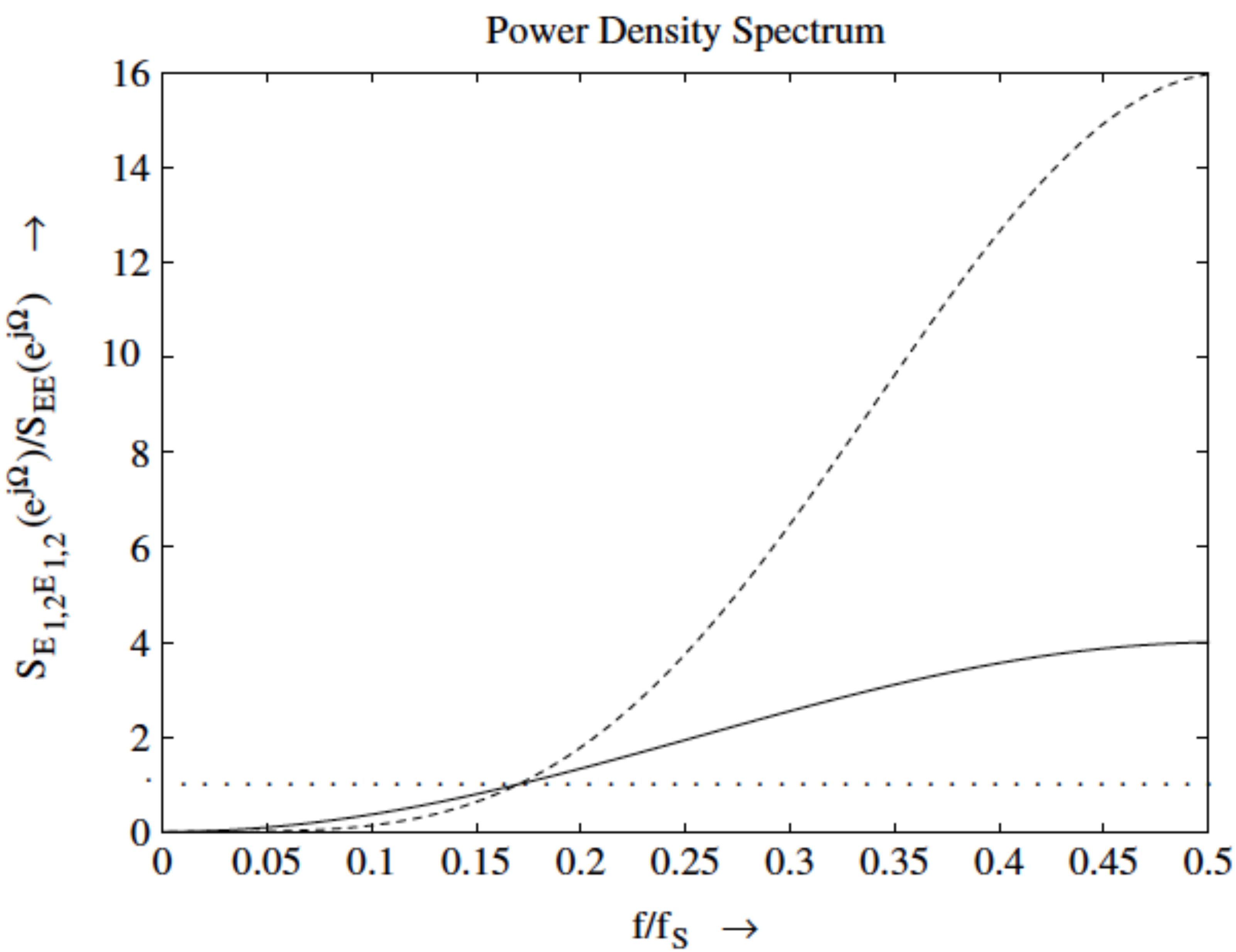
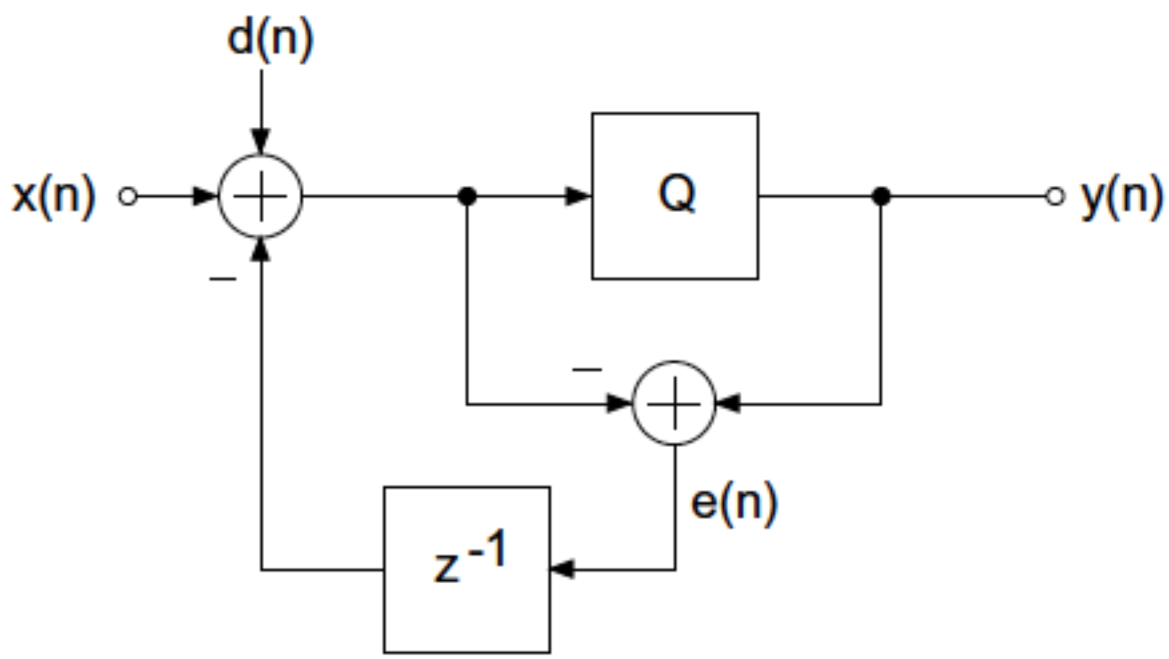


Figure 2.29 Spectrum shaping ($S_{EE}(e^{j\Omega})$ · · · , $S_{E_1E_1}(e^{j\Omega})$ —, $S_{E_2E_2}(e^{j\Omega})$ - - -).

DITHER + NOISE SHAPING

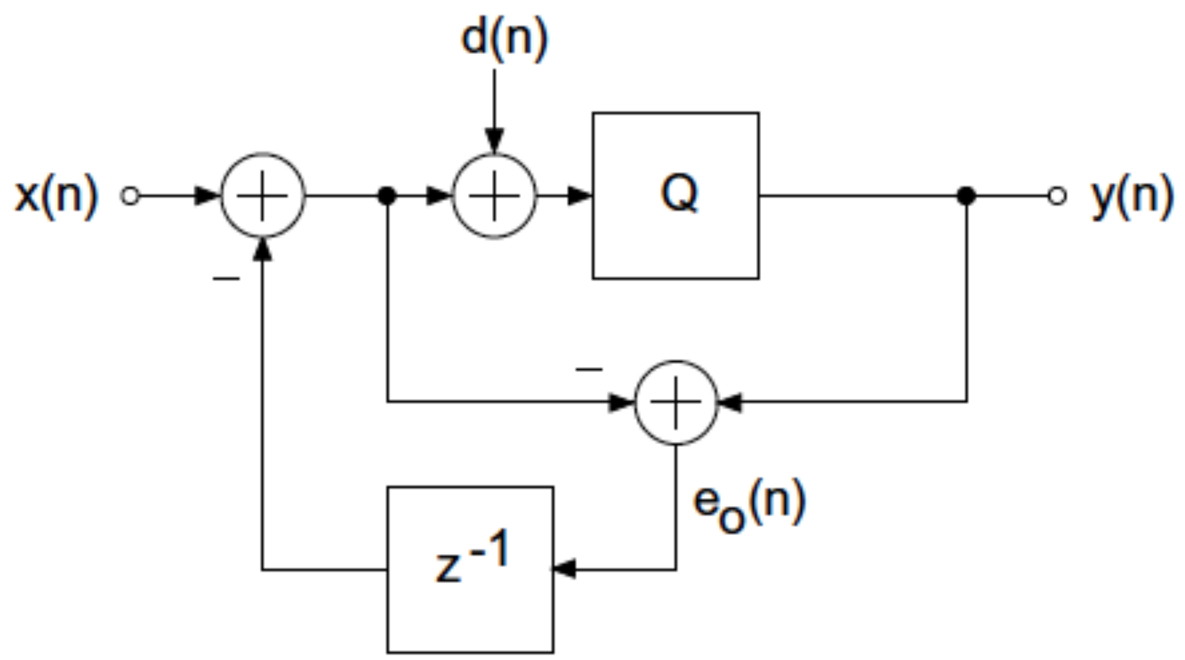


$$
\begin{aligned}
y(n) &= [x(n) + d(n) - e(n-1)]_Q \\
&= x(n) + d(n) - e(n-1) + e(n)
\end{aligned}
$$

$$
\begin{aligned}
e_1(n) &= y(n) - x(n) \\
&= d(n) + e(n) - e(n-1).
\end{aligned}
$$

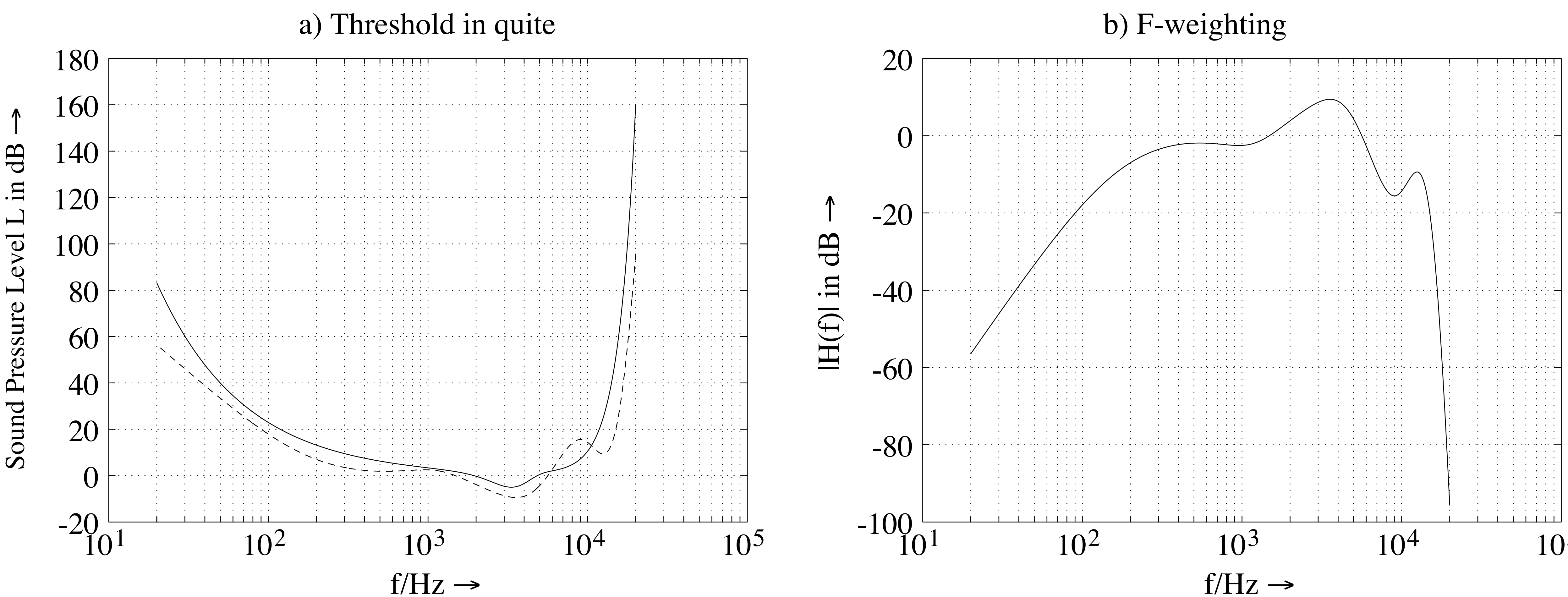
$$
\begin{aligned}
Y(z) &= X(z) + E(z)[1 - z^{-1}] + D(z) \\
\Rightarrow \quad E_1(z) &= E(z)[1 - z^{-1}] + D(z).
\end{aligned}
$$

MODIFIED NOISE SHAPING WITH DITHER



$$
\begin{aligned}
y(n) &= [x(n) + d(n) - e_0(n-1)]_Q \\
&= x(n) + d(n) - e_0(n-1) + e(n) \\
e_0(n) &= y(n) - [x(n) - e_0(n-1)] \\
&= d(n) + e(n) \\
y(n) &= x(n) + d(n) - d(n-1) + e(n) - e(n-1) \\
e_1(n) &= d(n) - d(n-1) + e(n) - e(n-1) \\
Y(z) &= X(z) + E(z)[1 - z^{-1}] + D(z)[1 - z^{-1}] \\
\Rightarrow E_1(z) &= E(z)[1 - z^{-1}] + D(z)[1 - z^{-1}].
\end{aligned}
$$

PSYCHO-ACOUSTIC NOISE SHAPING



PSYCHO-ACOUSTIC NOISE SHAPING