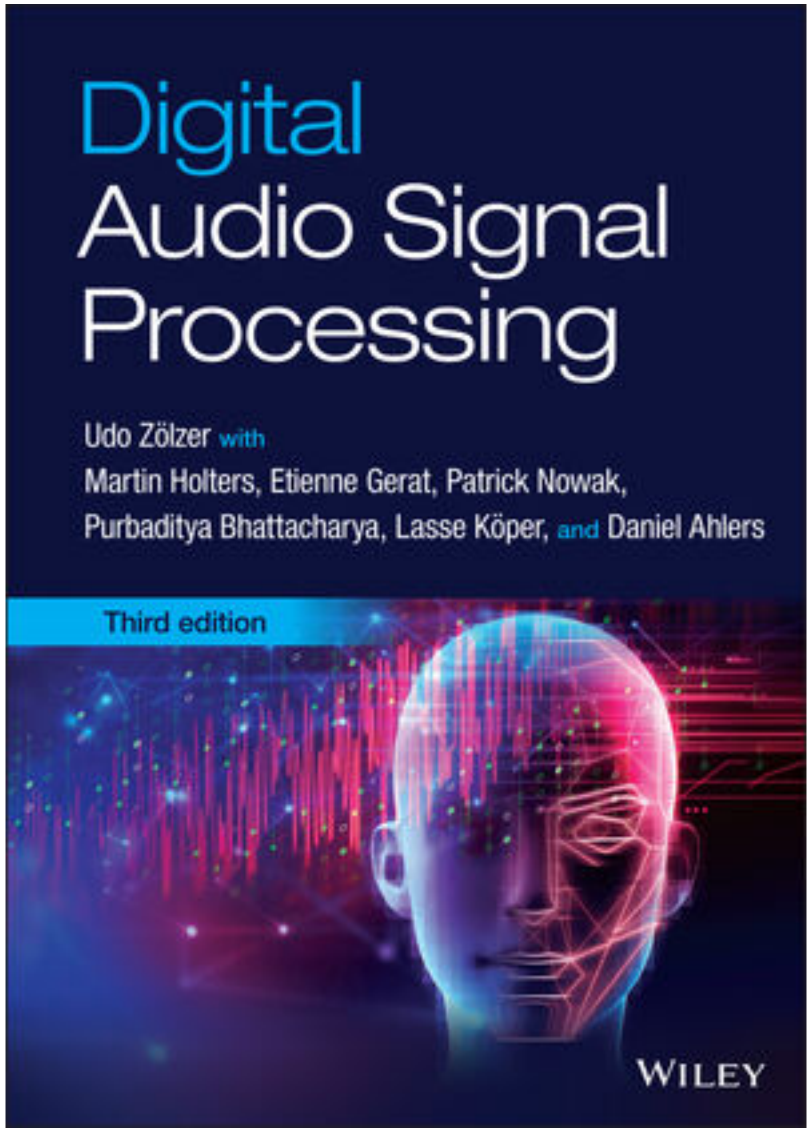


Lecture by Udo Zölzer

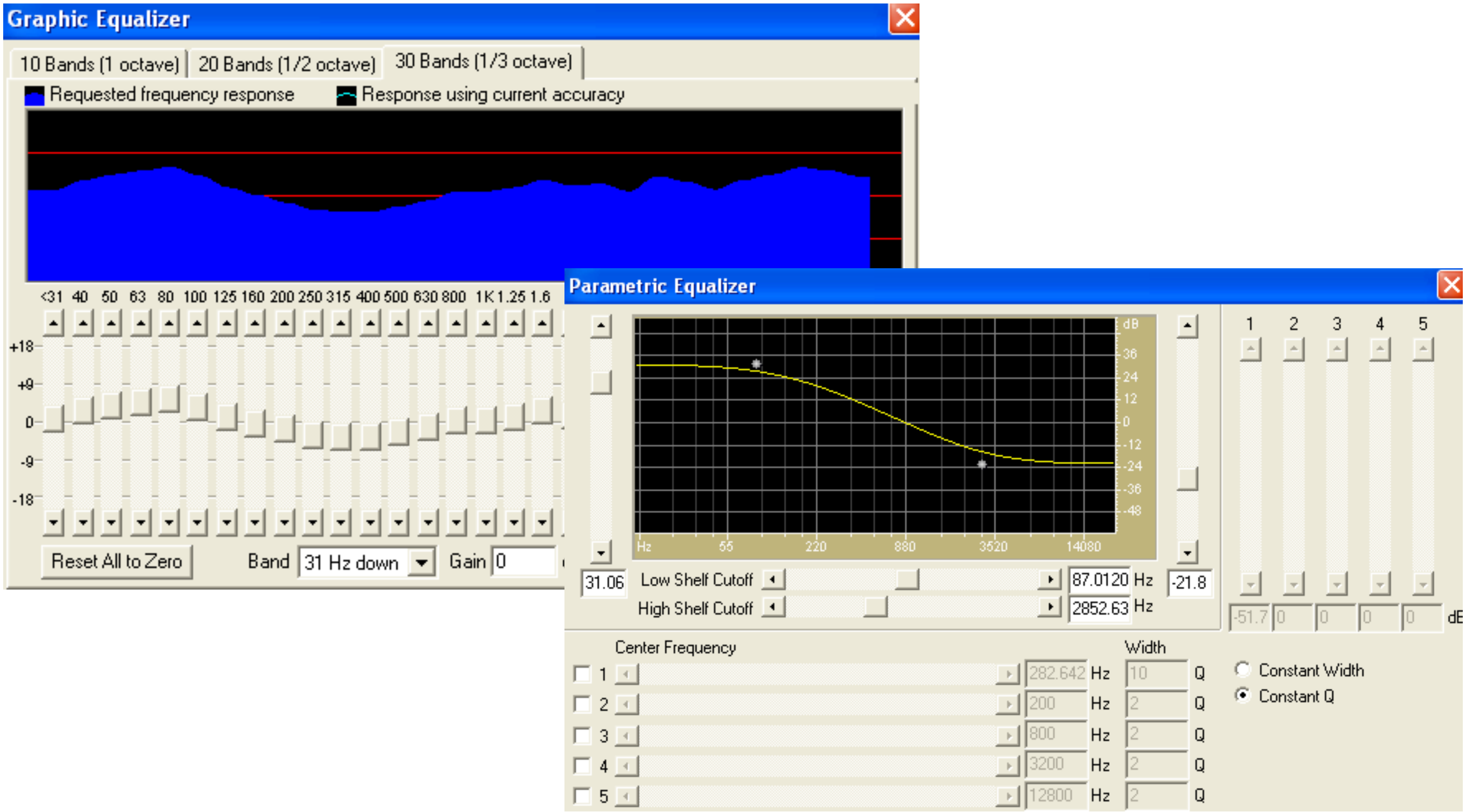
- Introduction
- Quantization
- Sampling Rate Conversion
- AD/DA Conversion
- Equalizers
- Room Simulation
- Dynamic Range Control
- Audio Coding
- Nonlinear Processing
- Machine Learning for Audio

RECURSIVE EQUALIZERS

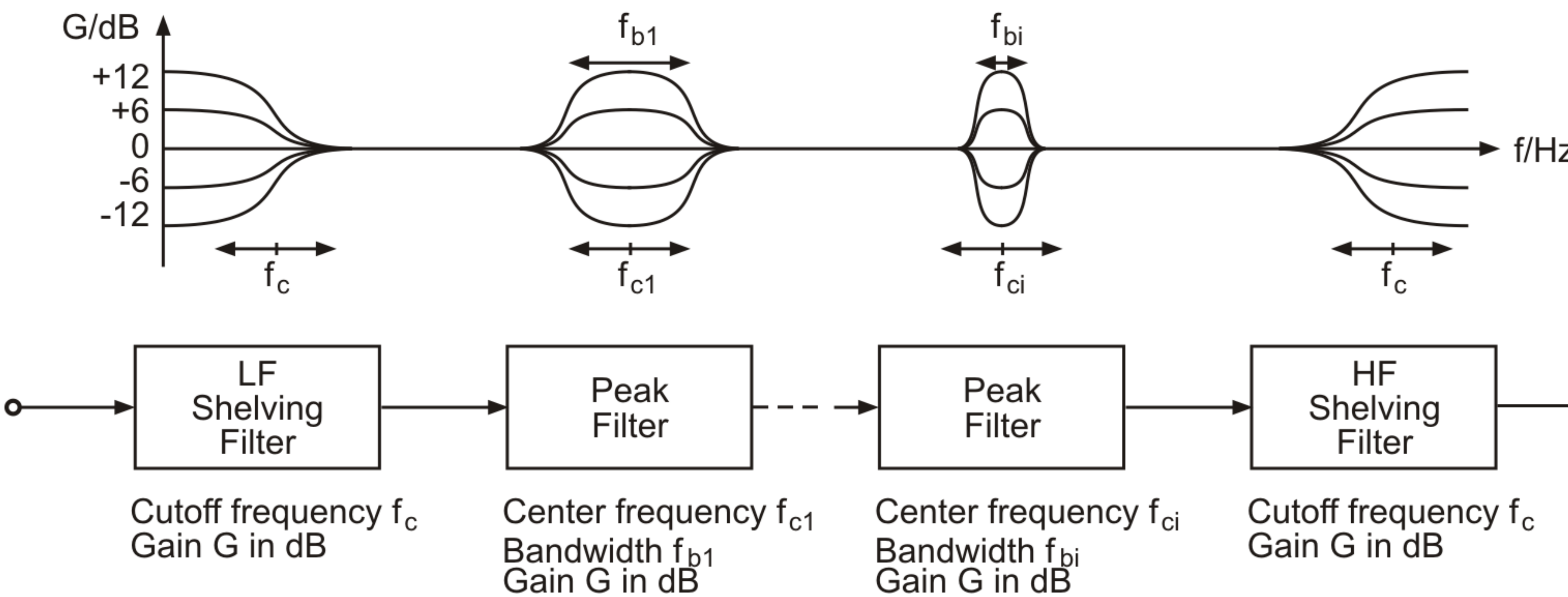
OUTLINE

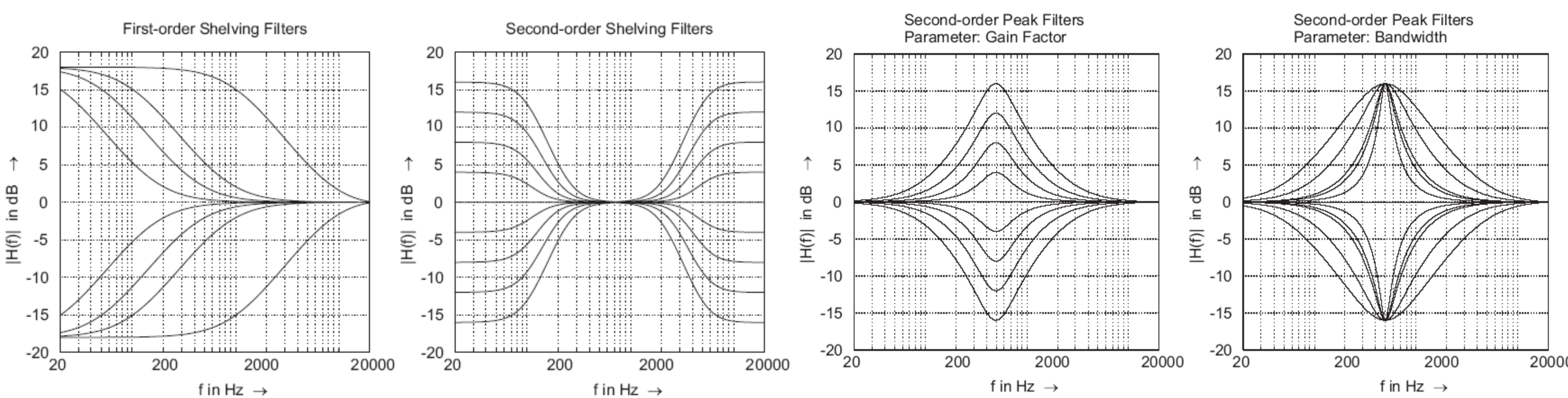
- Introduction
- Basics
    - LP, HP, and BP Filters with All-passes
    - Shelving and Peak Filters with All-passes
    - Filter Design for Shelving and Peak Filters
- Applications

AUDIO EQUALIZERS



EQUALIZER FREQUENCY RESPONSES





DESIGN AND IMPLEMENTATION OF SHELVING AND PEAK FILTERS

- Introduction
- Basics
  - LP, HP, and BP Filters with All-passes
  - Shelving and Peak Filters with All-passes
  - Filter Design for Shelving and Peak Filters
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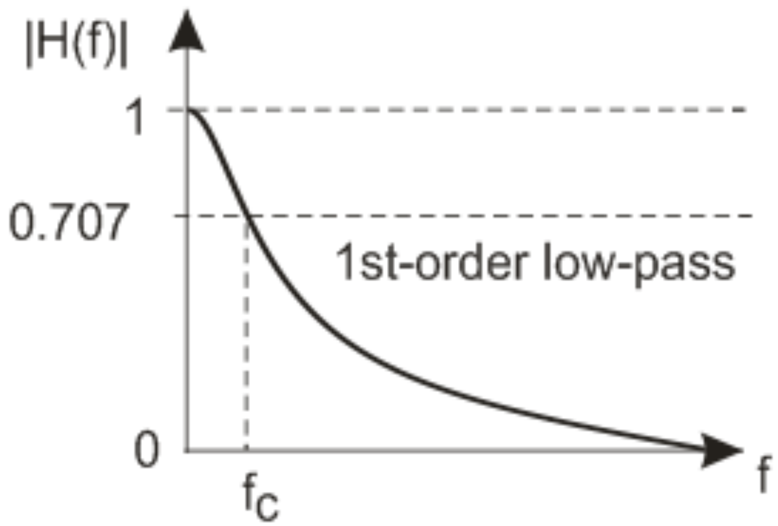
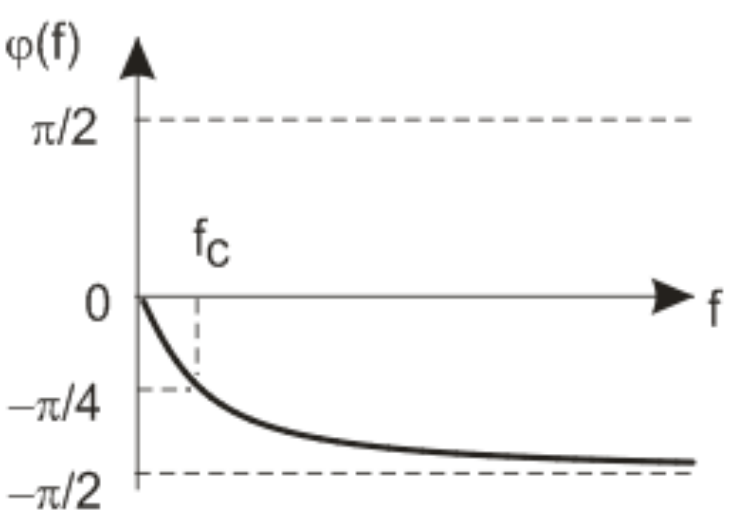
LP, HP, AND BP FILTERS

$$H_{LP}(z) = K_{LP}\frac{1+z^{-1}}{1-az^{-1}}$$

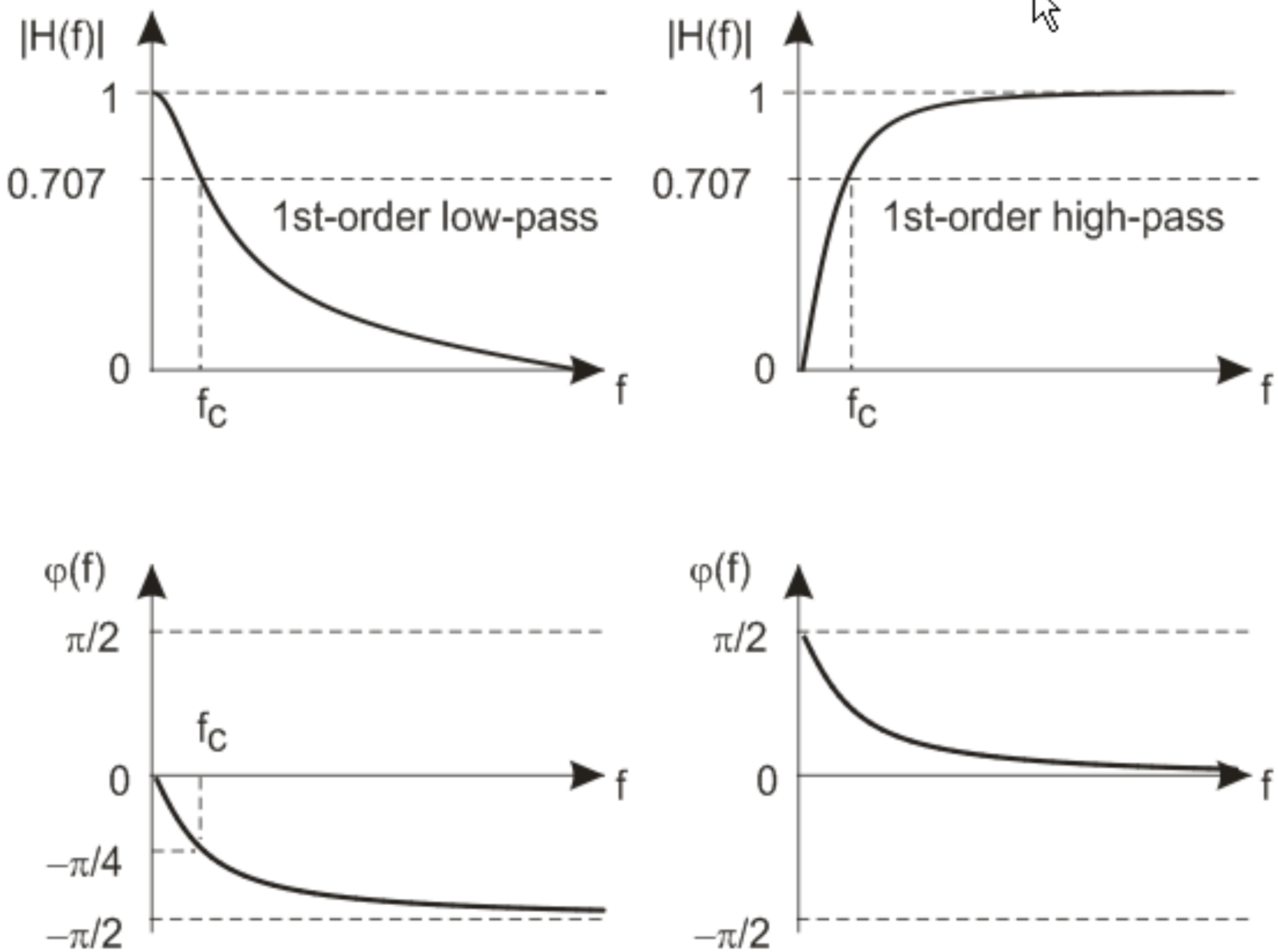




$$H(e^{j\Omega}) = H(z)|_{z=e^{j\Omega}} = |H(e^{j\Omega})|e^{j\varphi(\Omega)} \text{ with } \Omega = 2\pi fT \rightarrow |H(f)|e^{j\varphi(f)}$$

LP, HP, AND BP FILTERS

$$H_{LP}(z) = K_{LP}\frac{1+z^{-1}}{1-az^{-1}} \qquad H_{HP}(z) = K_{HP}\frac{1-z^{-1}}{1-az^{-1}}$$

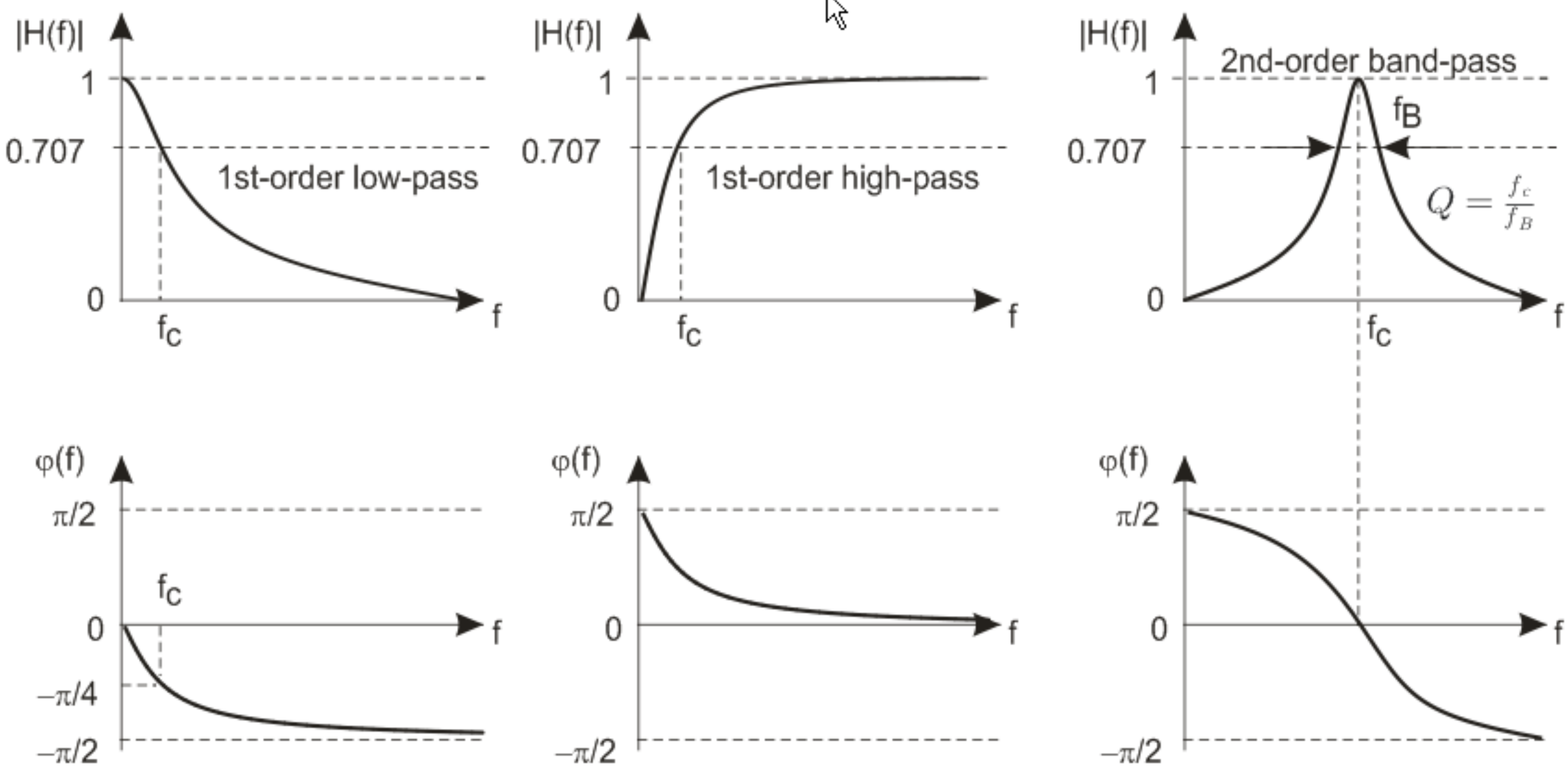


$$H(e^{j\Omega}) = H(z)|_{z=e^{j\Omega}} = |H(e^{j\Omega})|e^{j\varphi(\Omega)} \text{ with } \Omega = 2\pi fT \rightarrow |H(f)|e^{j\varphi(f)}$$

LP, HP, AND BP FILTERS

$$z^{-1} \Rightarrow -z^{-1}\frac{-b+z^{-1}}{1-bz^{-1}}$$

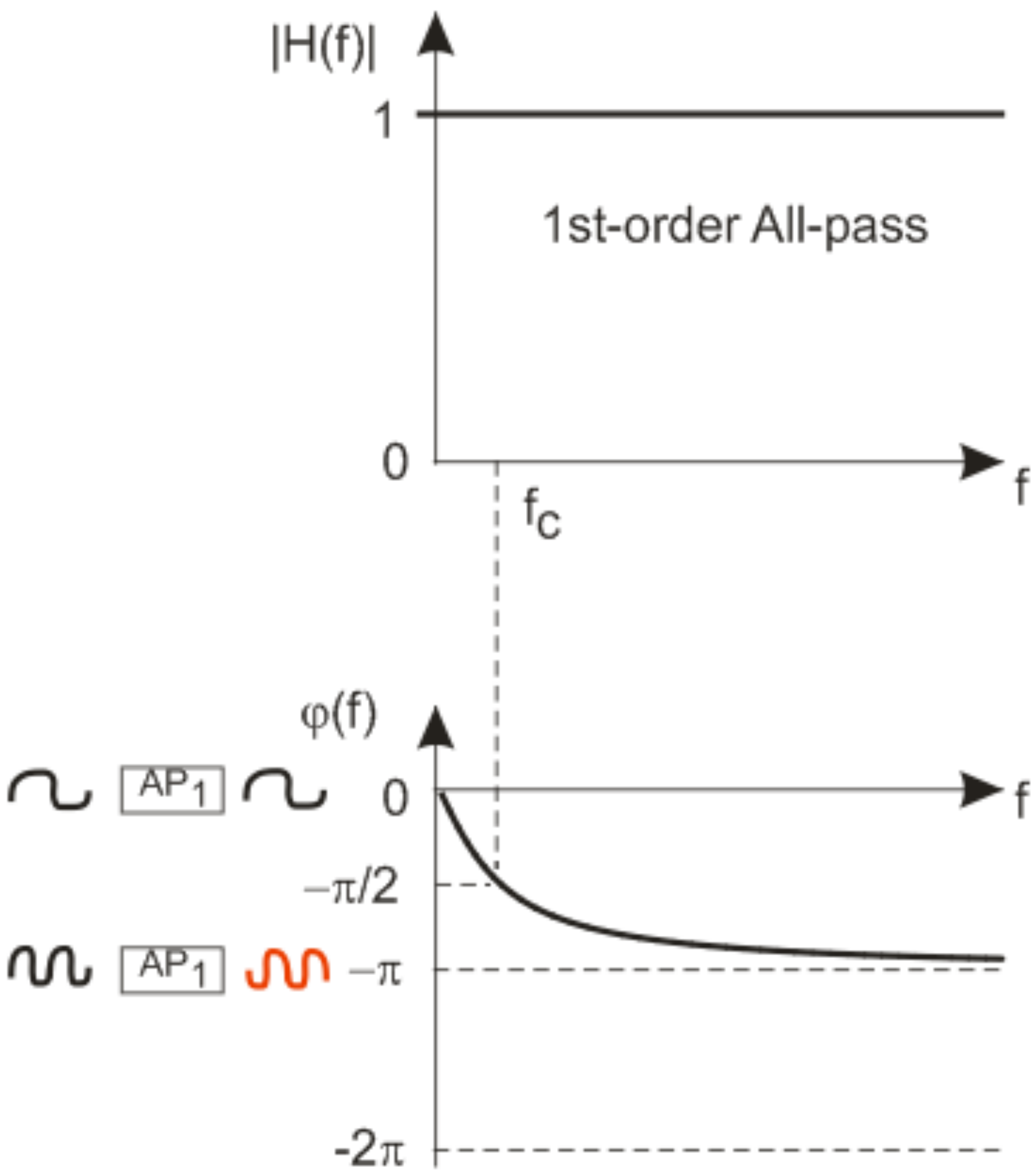
$H_{LP}(z) = K_{LP}\frac{1+z^{-1}}{1-az^{-1}}$ $H_{HP}(z) = K_{HP}\frac{1-z^{-1}}{1-az^{-1}}$ $H_{BP}(z) = K_{BP}\frac{1-z^{-2}}{1-b(1+a)z^{-1}+az^{-2}}$



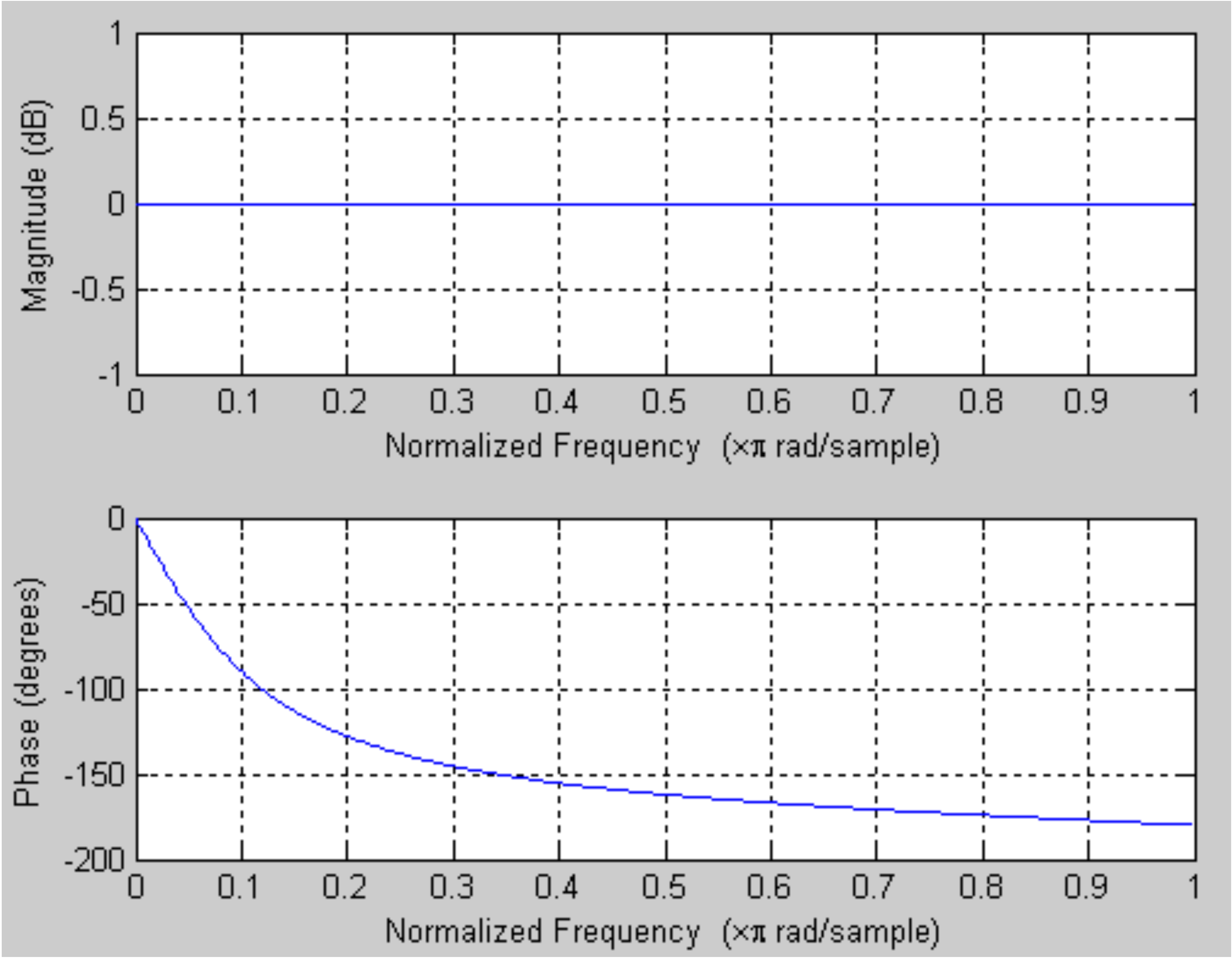
$$H(e^{j\Omega}) = H(z)|_{z=e^{j\Omega}} = |H(e^{j\Omega})|e^{j\varphi(\Omega)} \text{ with } \Omega = 2\pi fT \rightarrow |H(f)|e^{j\varphi(f)}$$

FIRST-ORDER ALL-PASS FILTERS

$$H_{AP_1}(z) = \frac{-a+z^{-1}}{1-az^{-1}} \quad a = \frac{1-\sin\Omega_c}{\cos\Omega_c}$$



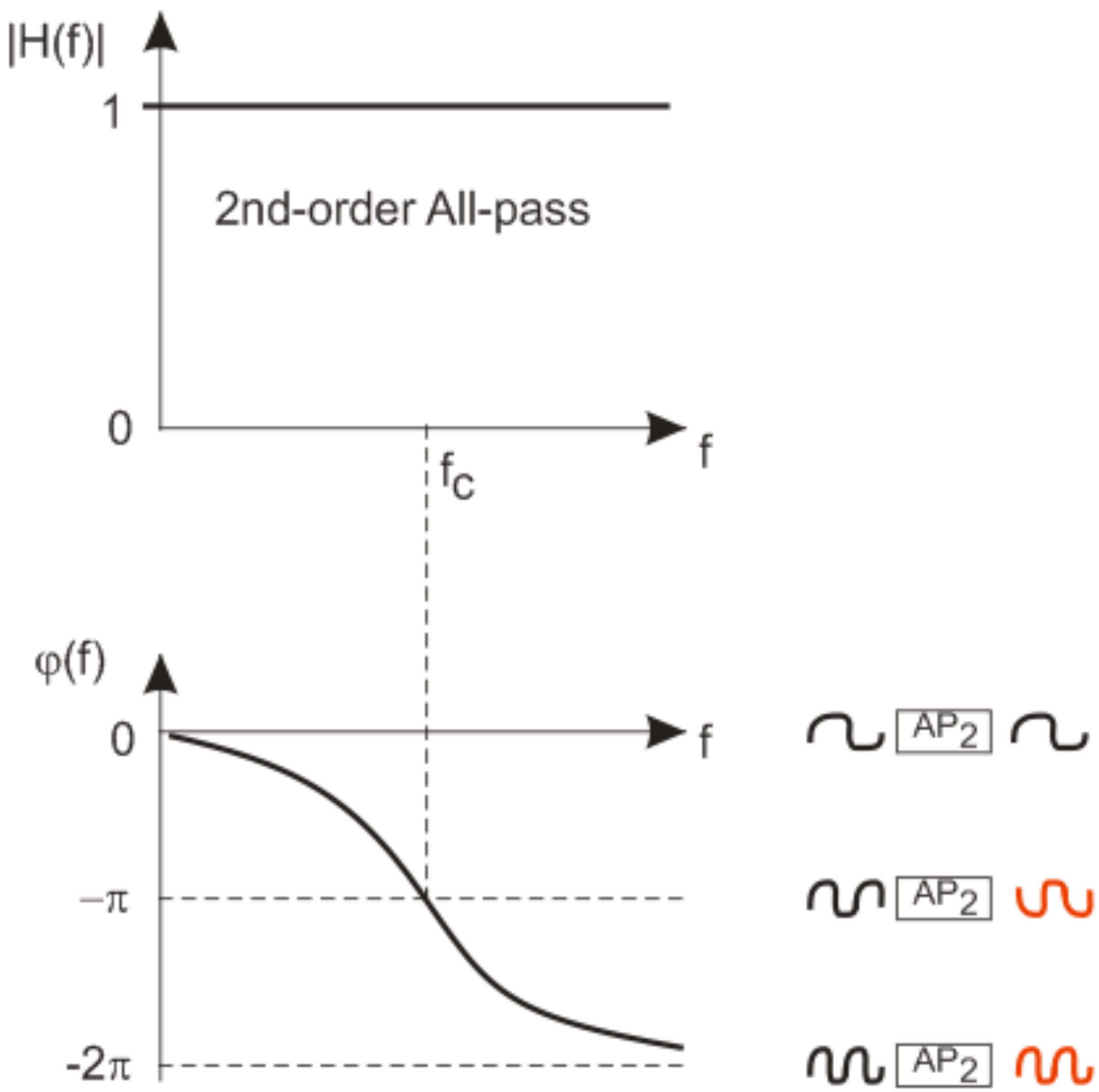

```
a=(1-sin(0.1*pi))/cos(0.1*pi);freqz([-a 1],[1 -a]);
```

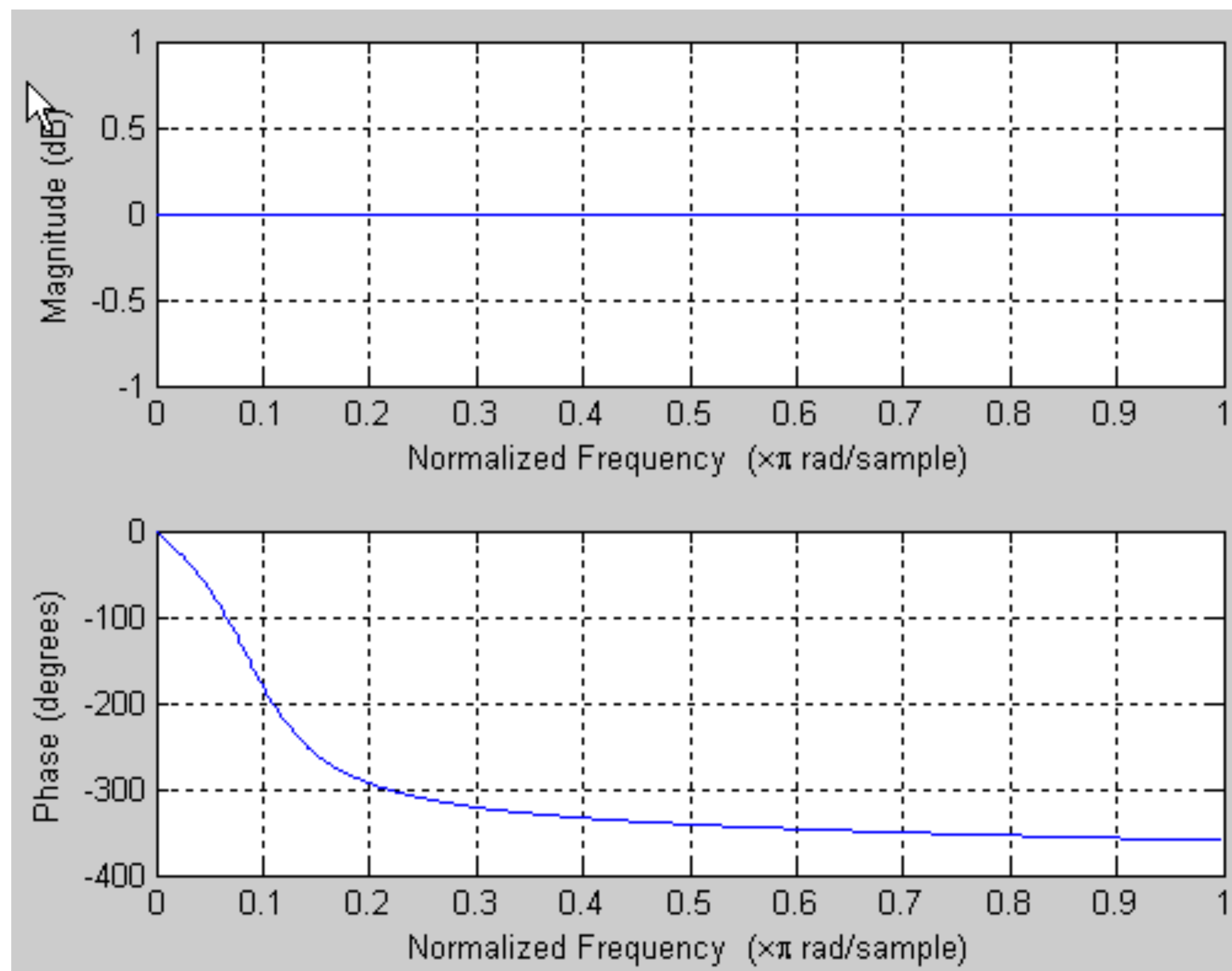


SECOND-ORDER ALL-PASS FILTERS

$$H_{AP_2}(z) = \frac{a - b(1+a)z^{-1} + z^{-2}}{1 - b(1+a)z^{-1} + az^{-2}} \quad b = \cos\Omega_c$$

```
freqz([a -b*(1+a) 1],[1 -b*(1+a) a])
```

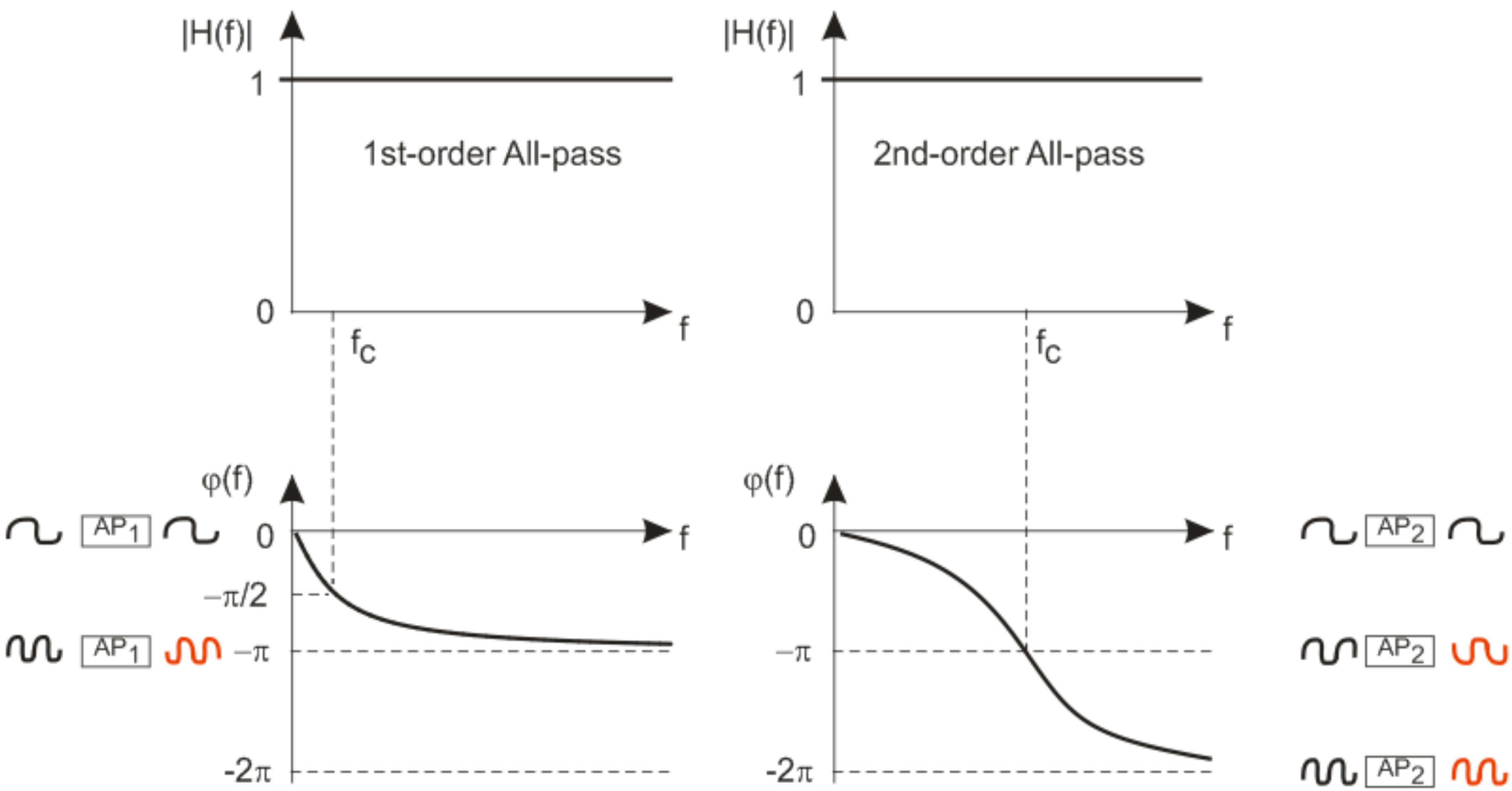




ALL-PASS FILTERS

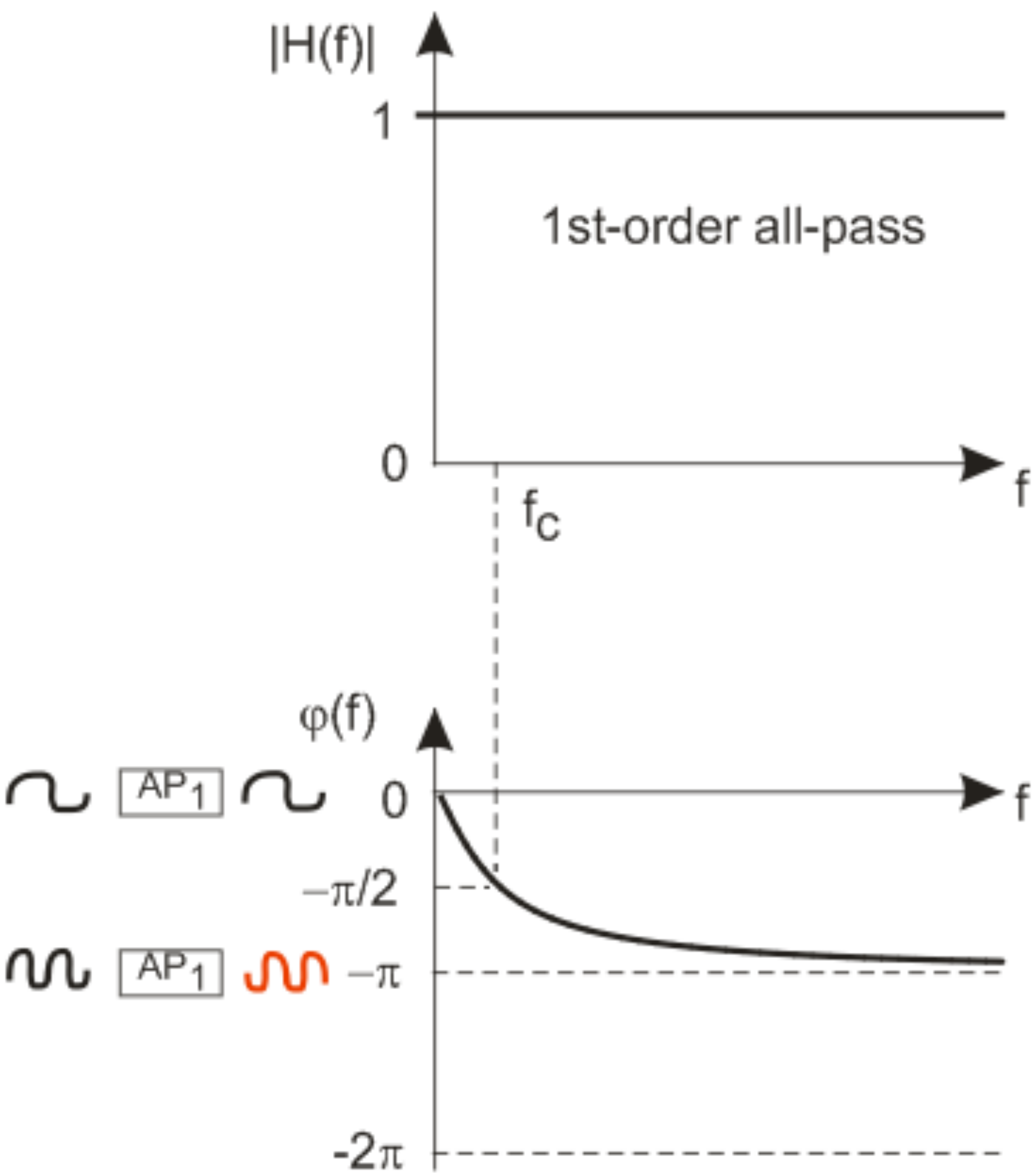
$$H_{AP_1}(z) = \frac{-a+z^{-1}}{1-az^{-1}} \quad a = \frac{1-\sin\Omega_c}{\cos\Omega_c}$$

$$H_{AP_2}(z) = \frac{a-b(1+a)z^{-1}+z^{-2}}{1-b(1+a)z^{-1}+az^{-2}} \quad b = \cos\Omega_c$$



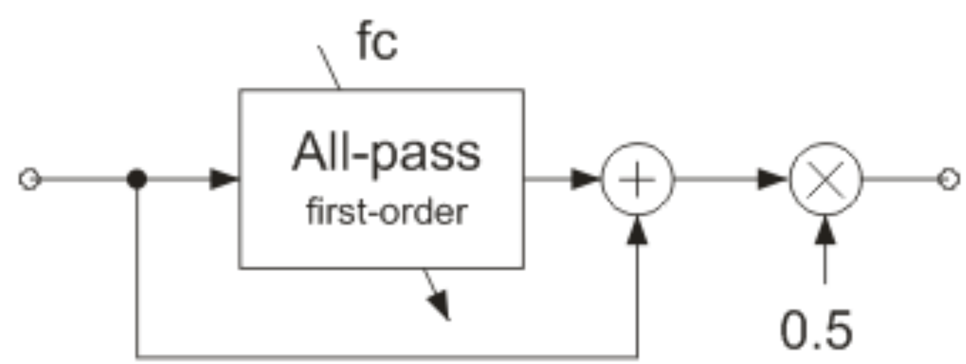
FIRST-ORDER LP WITH AP

$$H_{AP_1}(z) = \frac{-a+z^{-1}}{1-az^{-1}}$$



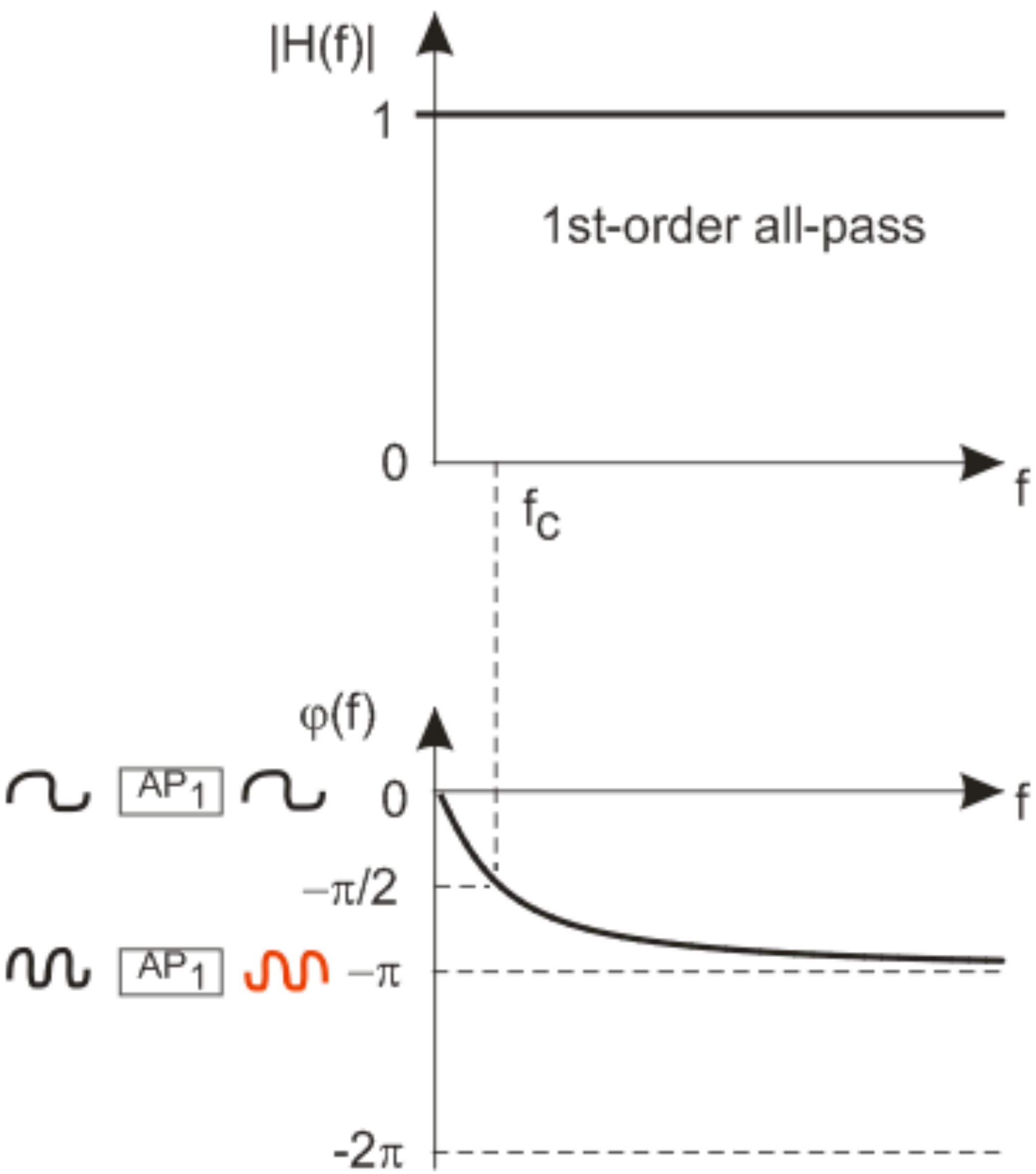
1st-order low-pass

$$H_{LP}(z) = 0.5 \cdot (1 + H_{AP_1}(z)) = 0.5 \cdot \left(1 + \frac{-a+z^{-1}}{1-az^{-1}}\right)$$



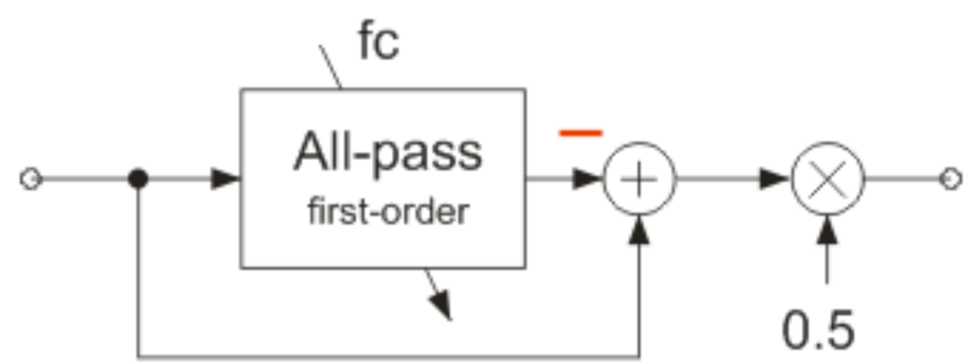
FIRST-ORDER HP WITH AP

$$H_{AP_1}(z) = \frac{-a+z^{-1}}{1-az^{-1}}$$



1st-order high-pass

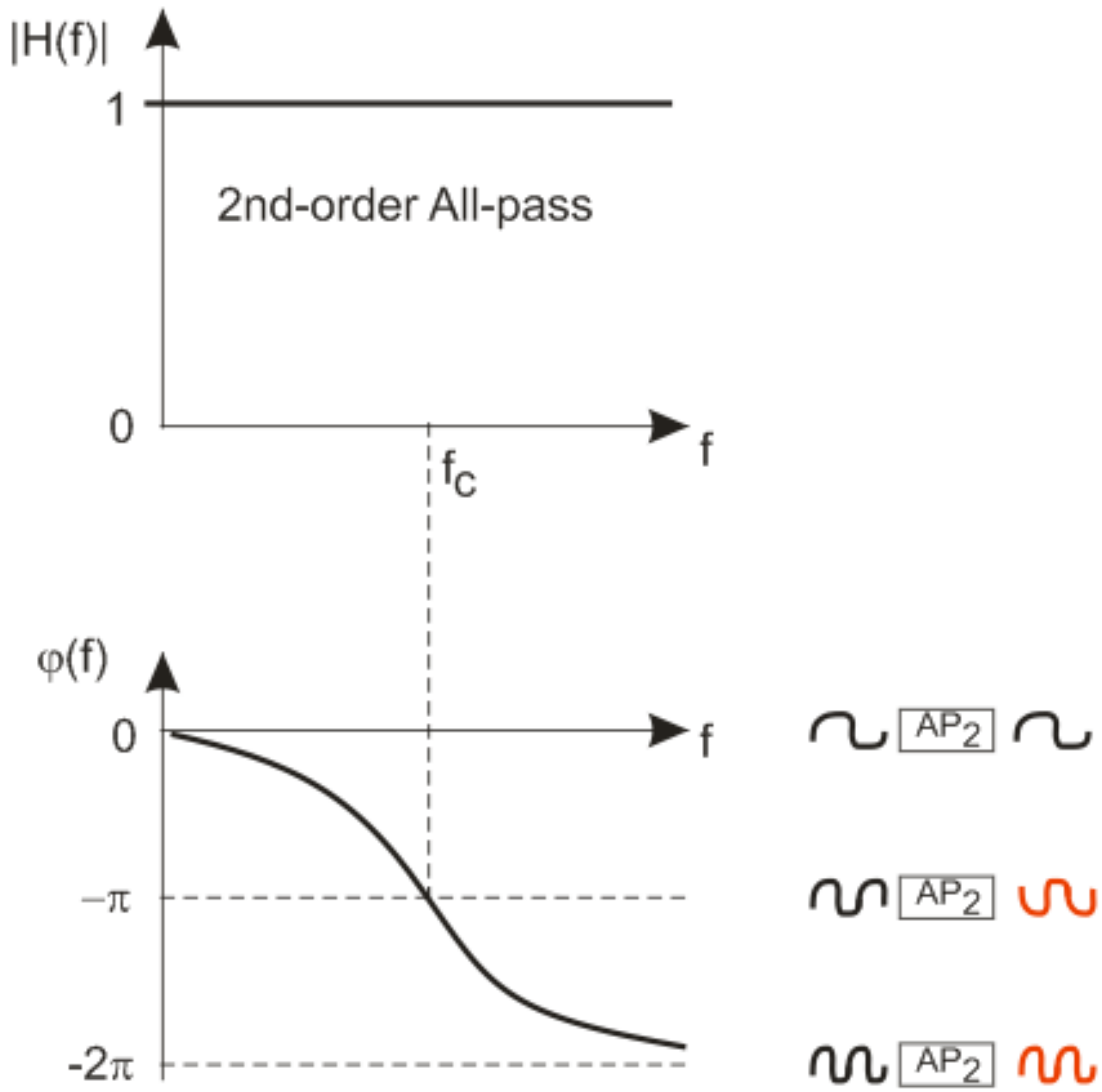
$$H_{HP}(z) = 0.5 \cdot (1 - H_{AP_1}(z)) = 0.5 \cdot \left(1 - \frac{-a+z^{-1}}{1-az^{-1}}\right)$$



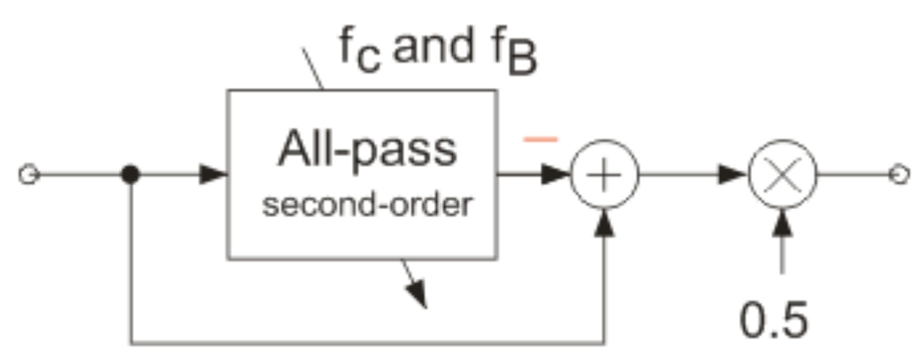
SECOND-ORDER BP WITH AP

$$H_{AP_2}(z) = \frac{a-b(1+a)z^{-1}+z^{-2}}{1-b(1+a)z^{-1}+az^{-2}}$$

$$H_{BP}(z) = 0.5\cdot\left(1-H_{AP_2}(z)\right) = 0.5\cdot\left(1-\frac{a-b(1+a)z^{-1}+z^{-2}}{1-b(1+a)z^{-1}+az^{-2}}\right)$$

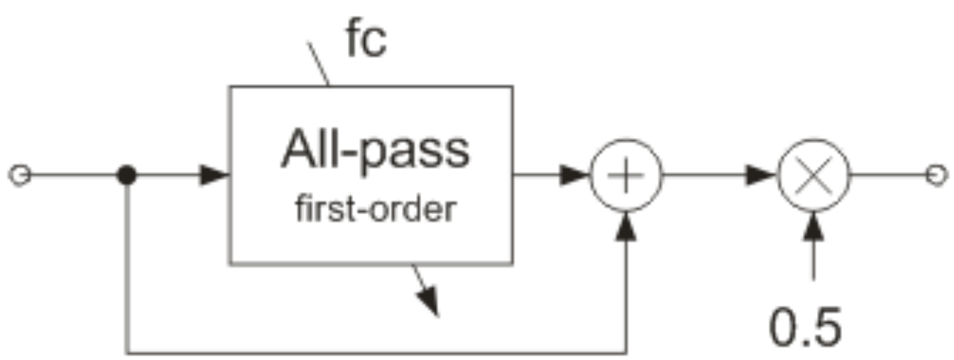


2nd-order band-pass

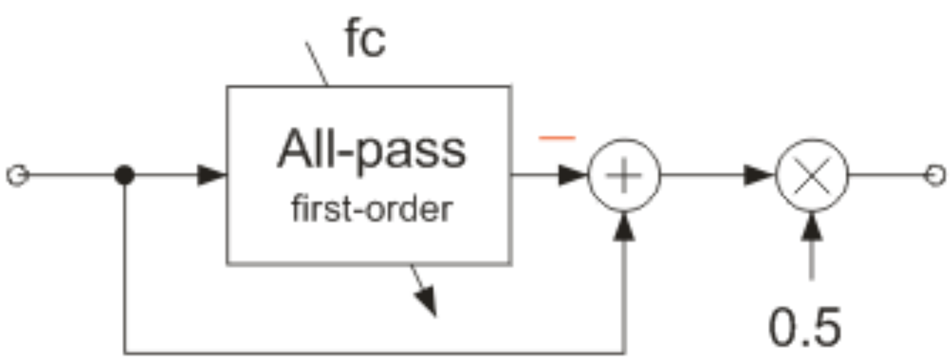


LP, HP, AND BP WITH AP

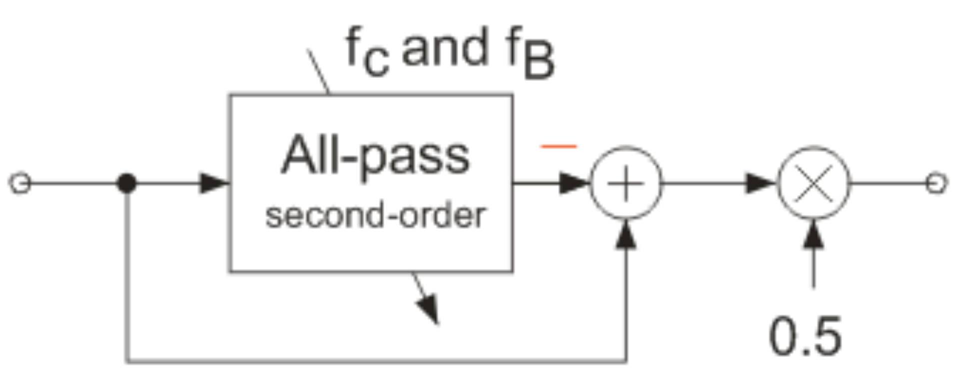
1st-order low-pass



1st-order high-pass



2nd-order band-pass



DESIGN AND IMPLEMENTATION OF SHELVING AND PEAK FILTERS

- Introduction
- Basics
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    - **Shelving and Peak Filters with All-passes**
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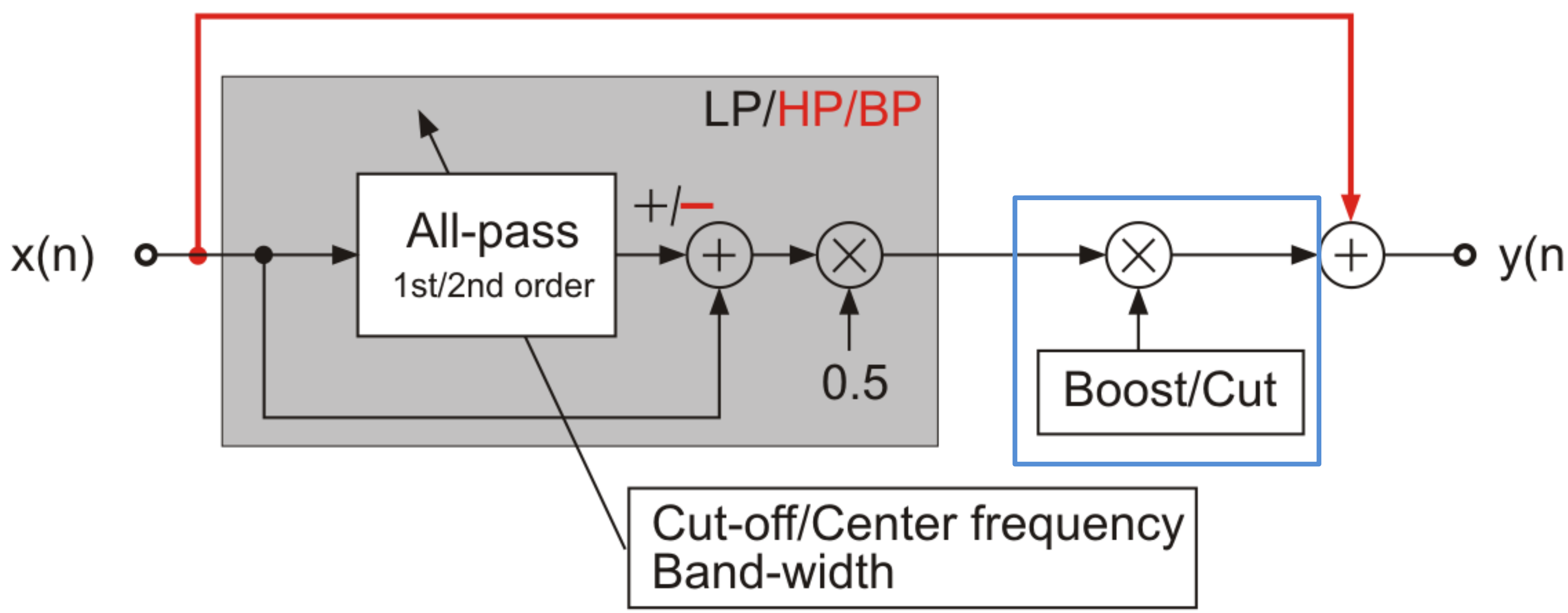
- Applications

SHELVING AND PEAK FILTERS WITH AP

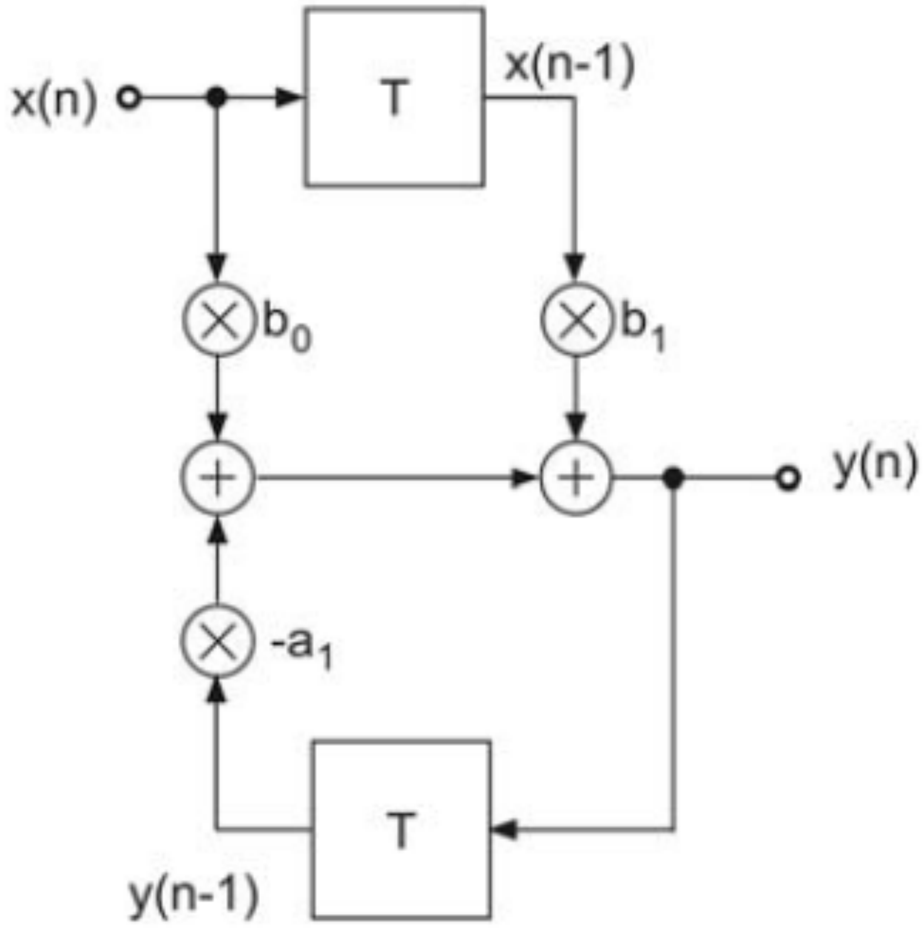


$$H_{LFS}(z) = 1 + B/C \cdot H_{LP}(z)$$

$$H_{HFS}(z) = 1 + B/C \cdot H_{HP}(z)$$

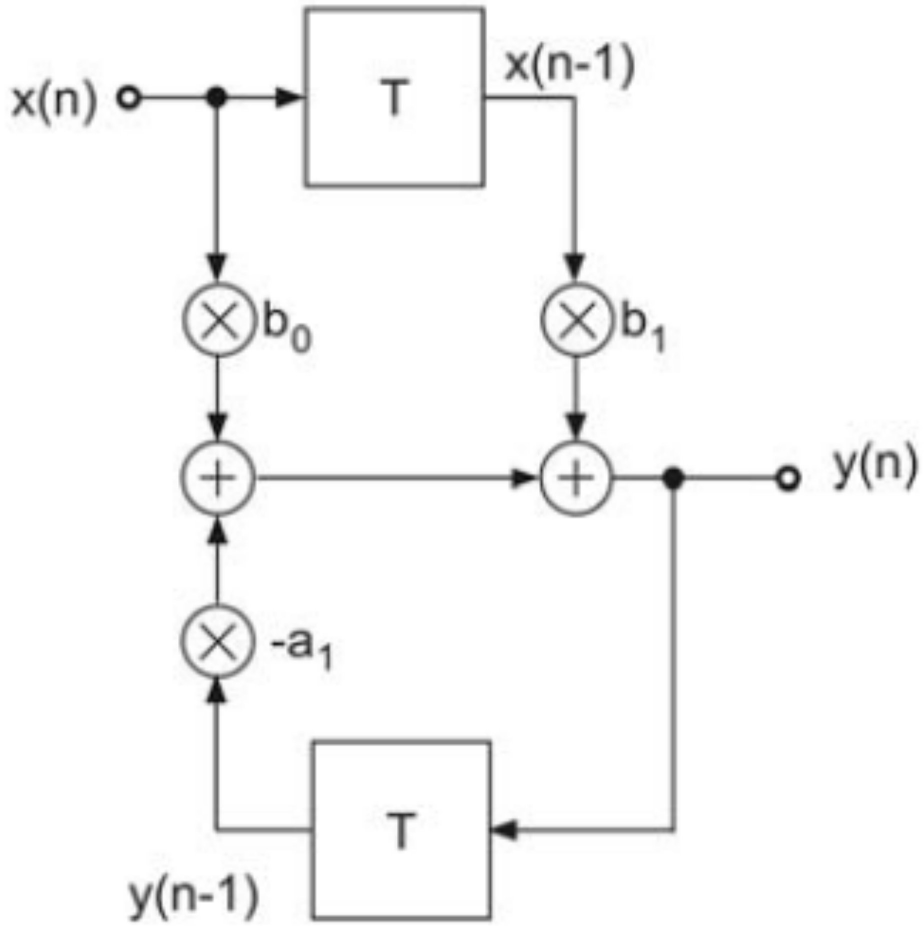
$$H_{PEAK}(z) = 1 + B/C \cdot H_{BP}(z)$$

FIRST-ORDER FILTERS

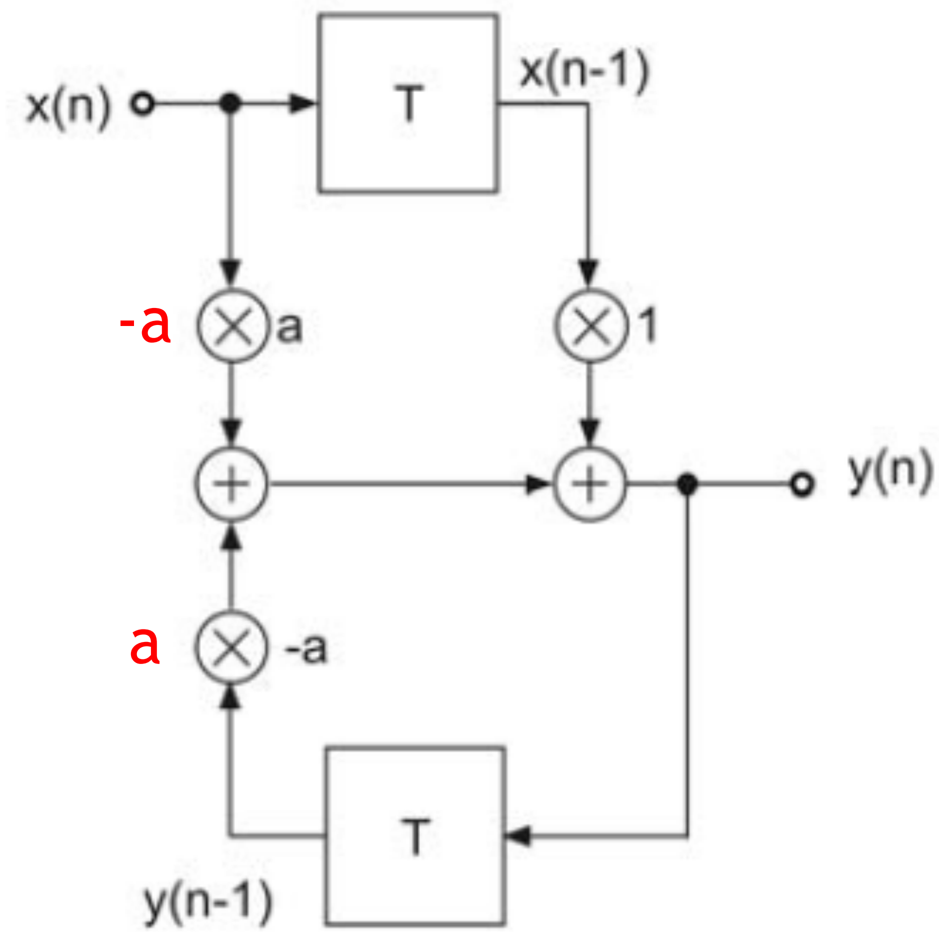


$$y(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) - a_1 \cdot y(n-1).$$

FIRST-ORDER FILTERS



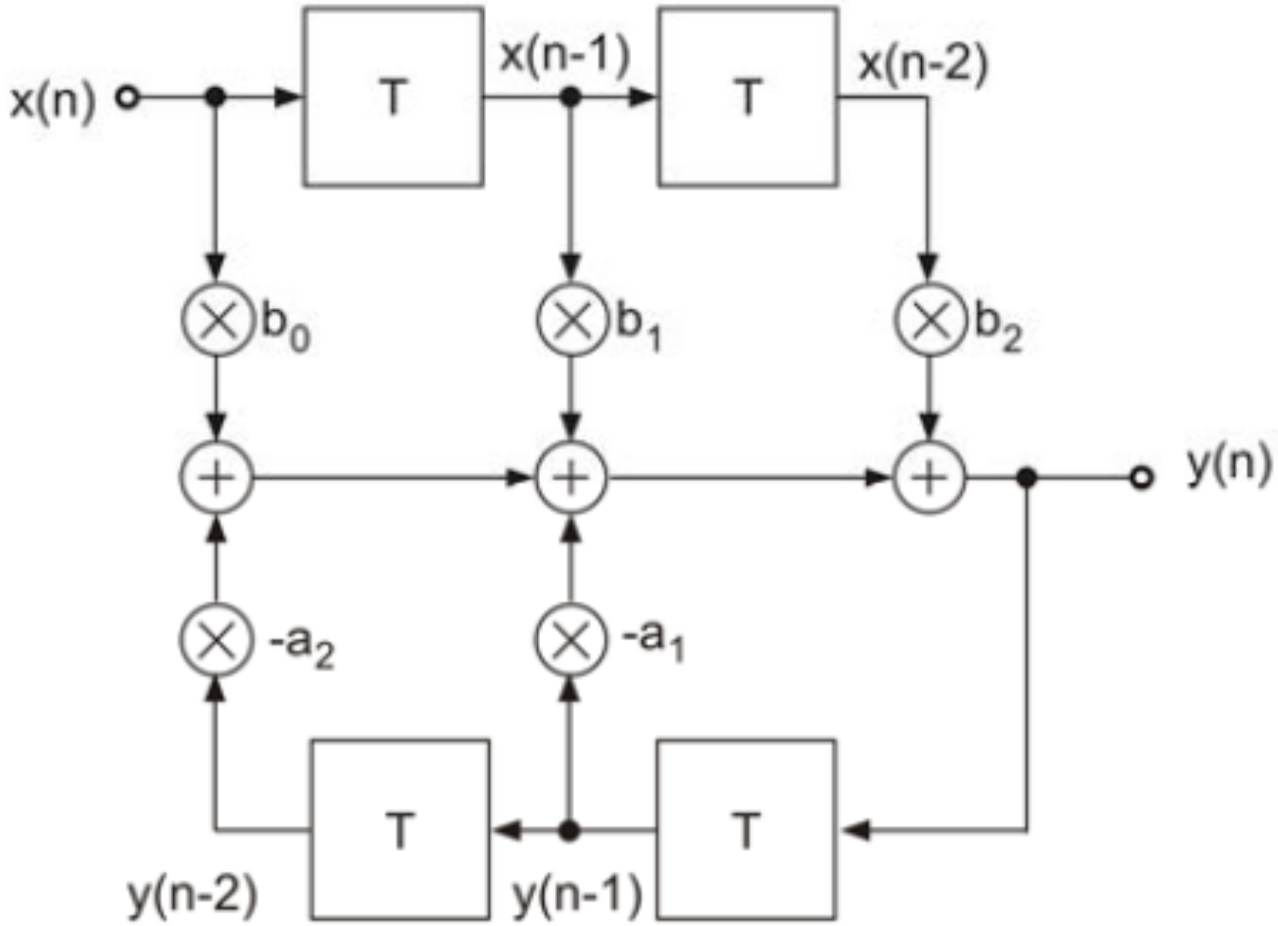
First-order All-pass



$$y(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) - a_1 \cdot y(n-1).$$

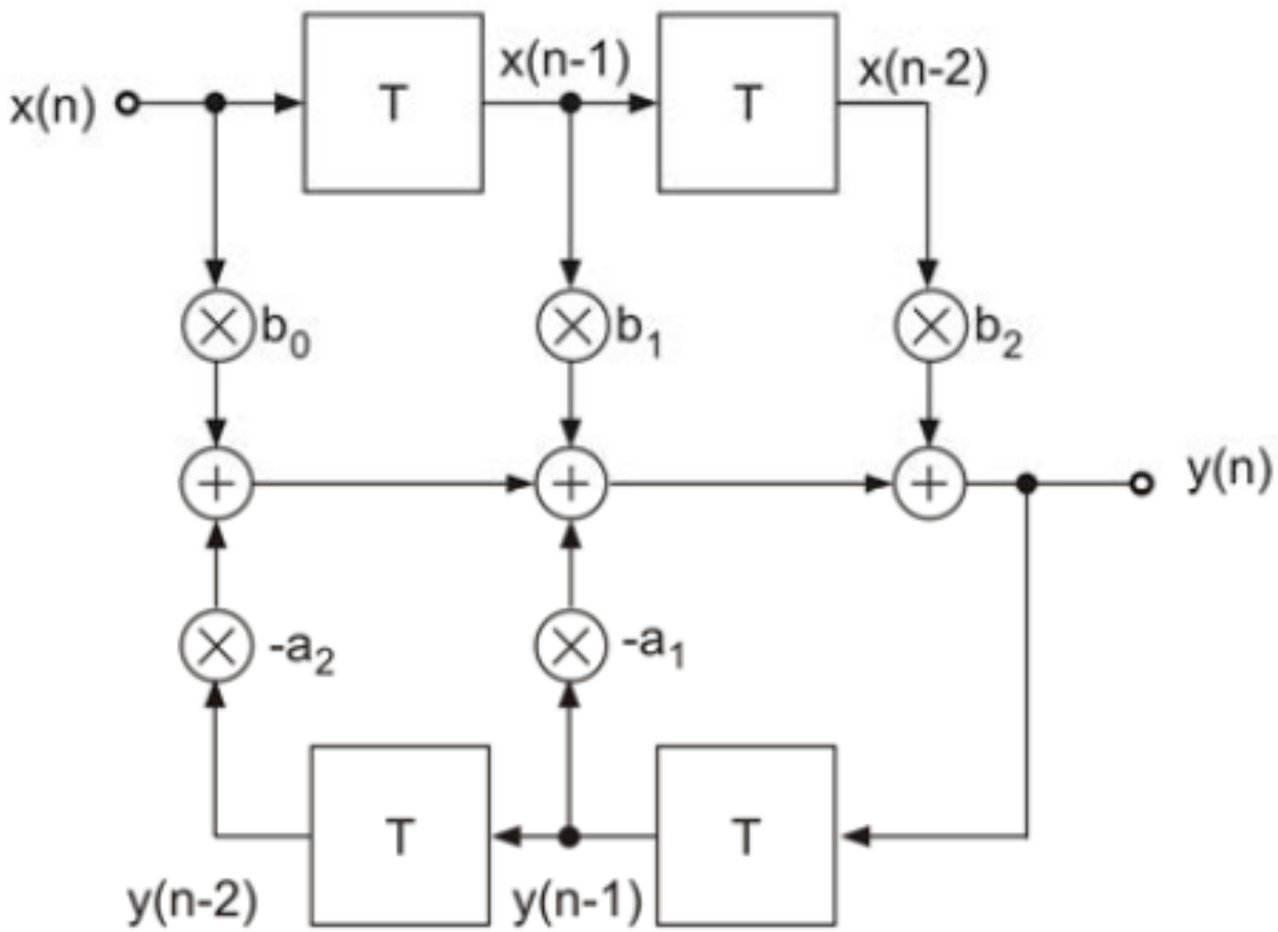
$$y(n) = x(n) * h_{AP1}(n) = a \cdot x(n) + x(n-1) - a \cdot y(n-1),$$

SECOND-ORDER FILTERS

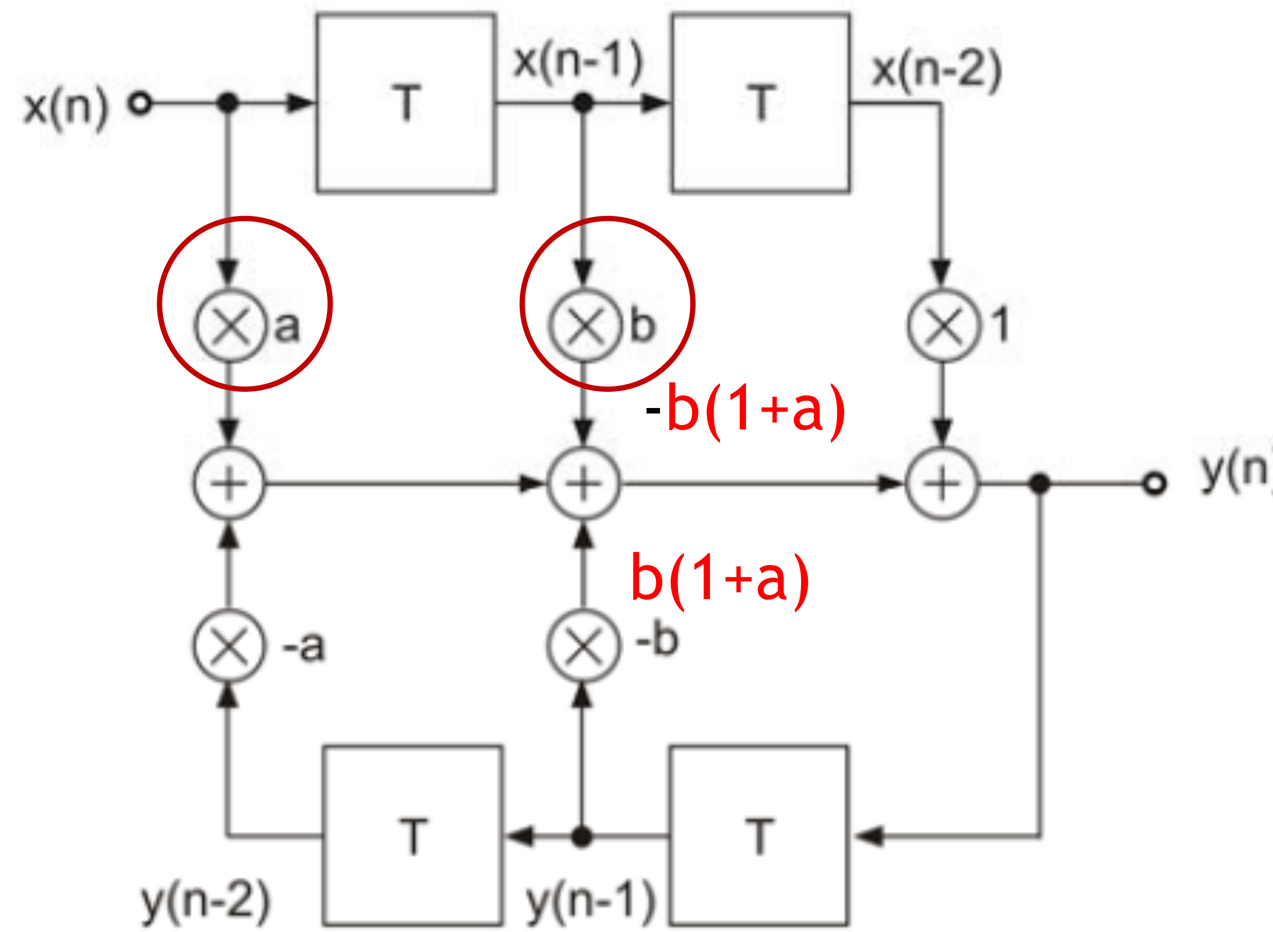


$$y(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) + b_2 \cdot x(n-2) - a_1 \cdot y(n-1) - a_2 \cdot y(n-2).$$

SECOND-ORDER FILTERS



Second-order All-pass



$$y(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) + b_2 \cdot x(n-2) - a_1 \cdot y(n-1) - a_2 \cdot y(n-2).$$

$$y(n) = x(n) * h_{AP2}(n).$$

$$= a \cdot x(n) + b \cdot x(n-1) + x(n-2) - b \cdot y(n-1) - a \cdot y(n-2).$$

DESIGN AND IMPLEMENTATION OF SHELVING AND PEAK FILTERS

- Introduction
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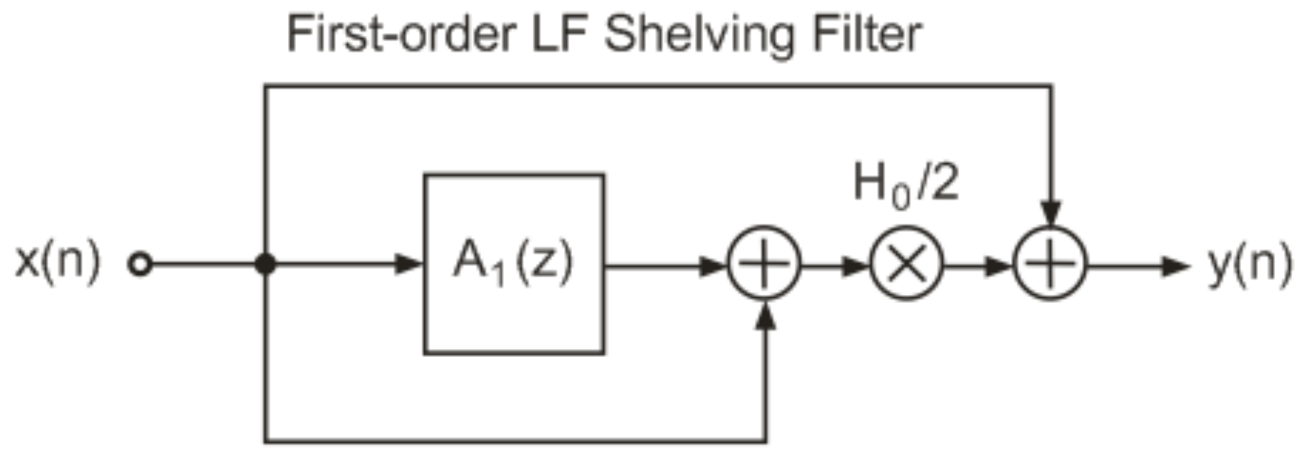
  - **Filter Design for Shelving and Peak Filters**
- Applications

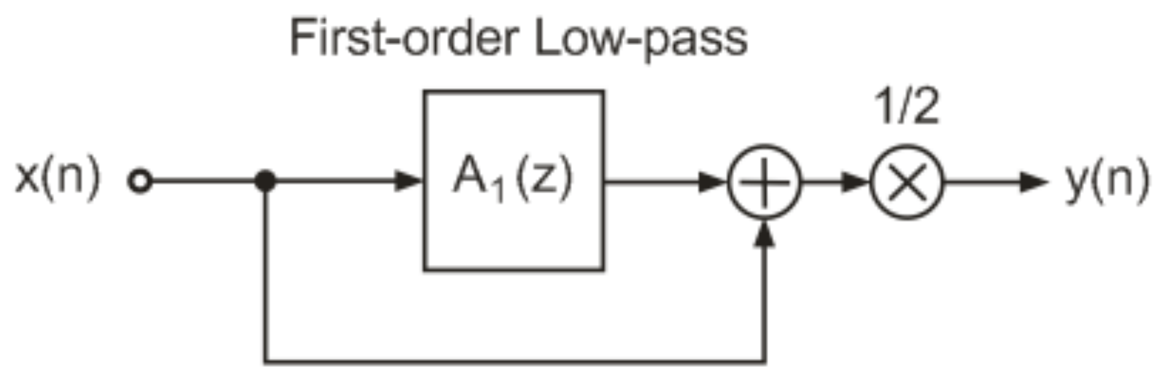
FILTER DESIGN – LF SHELVING FILTER

$$H(z) = 1 + \frac{H_0}{2}[1 + A_1(z)] \quad A_1(z) = \frac{-a+z^{-1}}{1-az^{-1}}$$

$$H(z=1) = V_0 = 1 + H_0, V_0 = 10^{G/20} \rightarrow H_0 = 10^{G/20} - 1$$



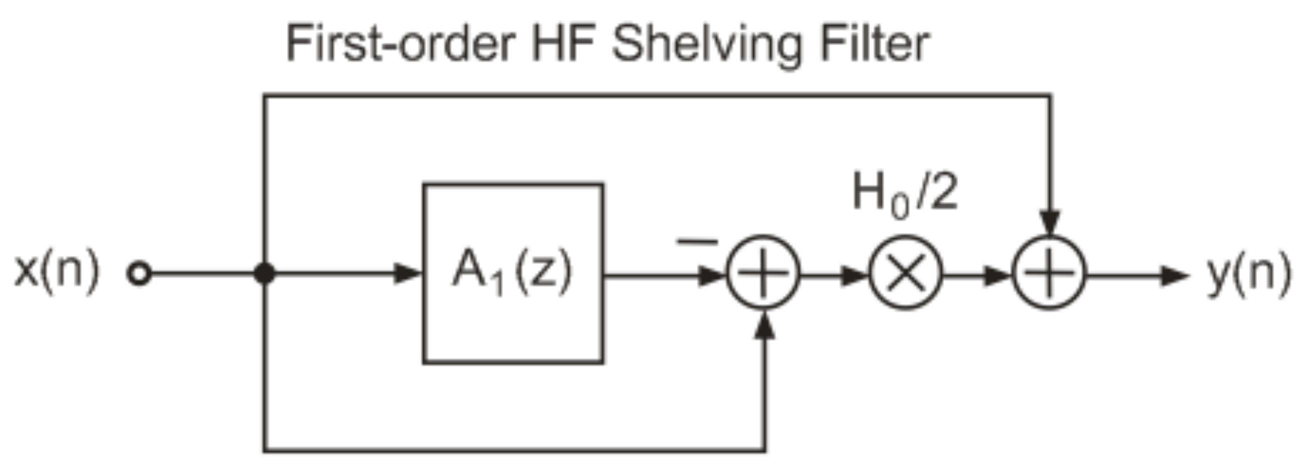
$$a_B = \frac{1-\tan\frac{\Omega_c}{2}}{1+\tan\frac{\Omega_c}{2}}, a_C = \frac{V_0-\tan\frac{\Omega_c}{2}}{V_0+\tan\frac{\Omega_c}{2}}$$



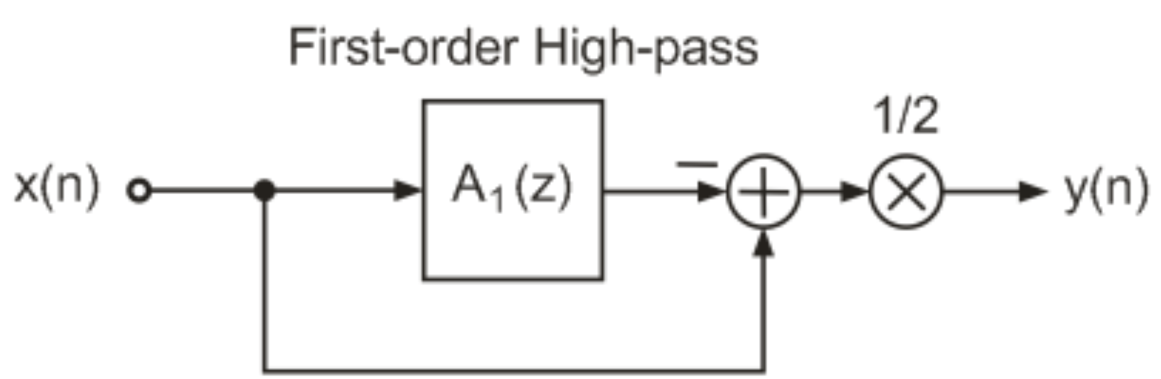
FILTER DESIGN – HF SHELVING FILTER

$$H(z) = 1 + \frac{H_0}{2}[1 - A_1(z)] \quad A_1(z) = \frac{-a+z^{-1}}{1-az^{-1}}$$

$$H(z=-1) = V_0 = 1 + H_0, V_0 = 10^{G/20} \rightarrow H_0 = 10^{G/20} - 1$$



$$a_B = \frac{1-\tan\frac{\Omega_c}{2}}{1+\tan\frac{\Omega_c}{2}}, \; a_C = \frac{1-V_0\tan\frac{\Omega_c}{2}}{1+V_0\tan\frac{\Omega_c}{2}}$$

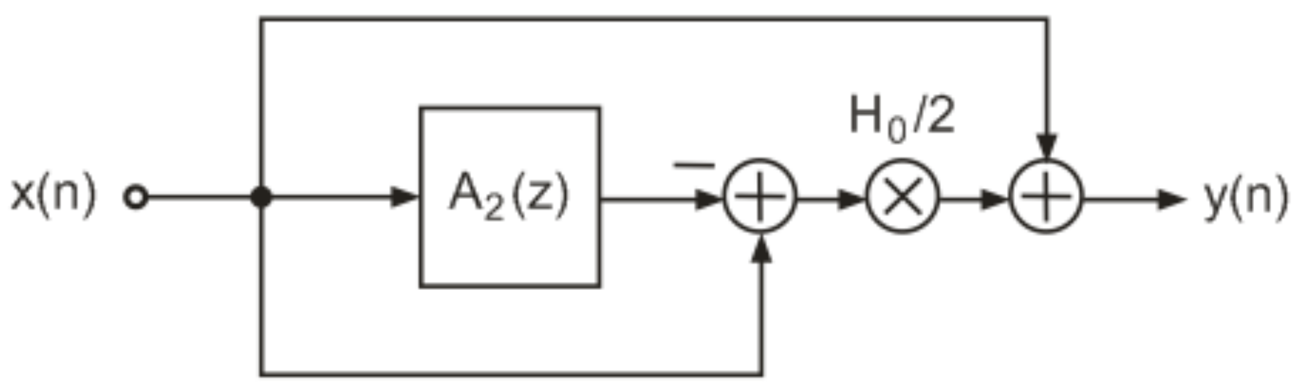


FILTER DESIGN – PEAK FILTER

$$H(z) = 1 + \frac{H_0}{2}[1 - A_2(z)] \quad A_2(z) = \frac{a - b(1+a)z^{-1} + z^{-2}}{1 - b(1+a)z^{-1} + az^{-2}}$$

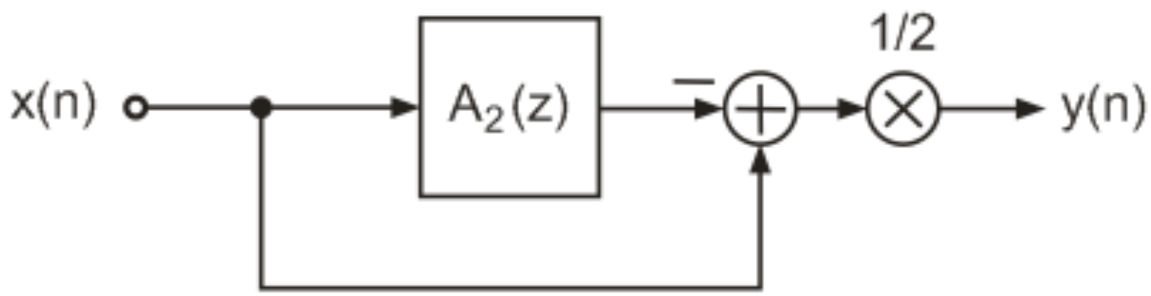
$$H(e^{j\Omega_c}) = V_0 = 1 + H_0, V_0 = 10^{G/20} \rightarrow H_0 = 10^{G/20} - 1$$

Second-order Peak Filter



$$b = \cos\Omega_c, \; a_B = \frac{1 - \tan\frac{\Omega_b}{2}}{1 + \tan\frac{\Omega_b}{2}}, \; a_C = \frac{V_0 - \tan\frac{\Omega_b}{2}}{V_0 + \tan\frac{\Omega_b}{2}}$$

Second-order Band-pass



APPLICATIONS

- All-pass realization of shelving and peak filters
- Flexible adaption of gain, bandwidth, cut-off frequency in real-time
- Adaptive equalizers
- Parametric filter structures
- Dynamic control of all filter parameters
- Extension to higher-order shelving and peak (see references)

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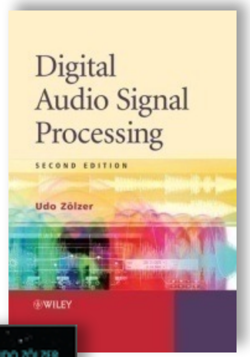

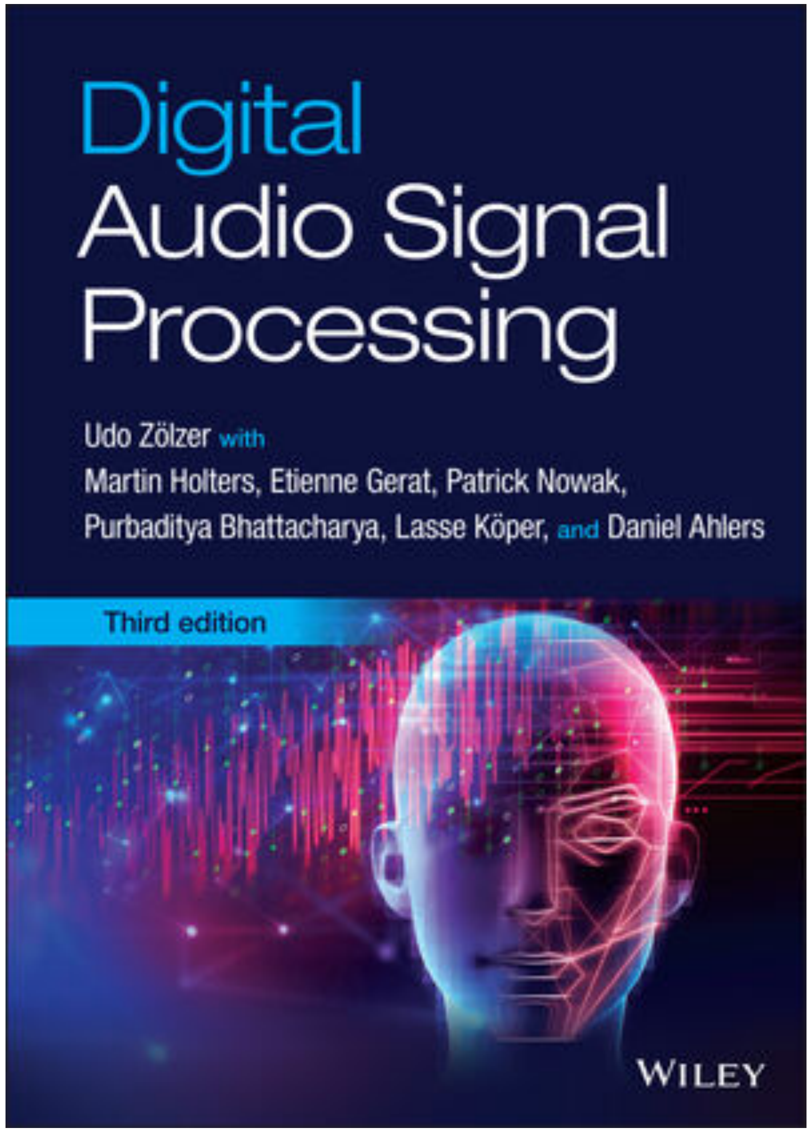

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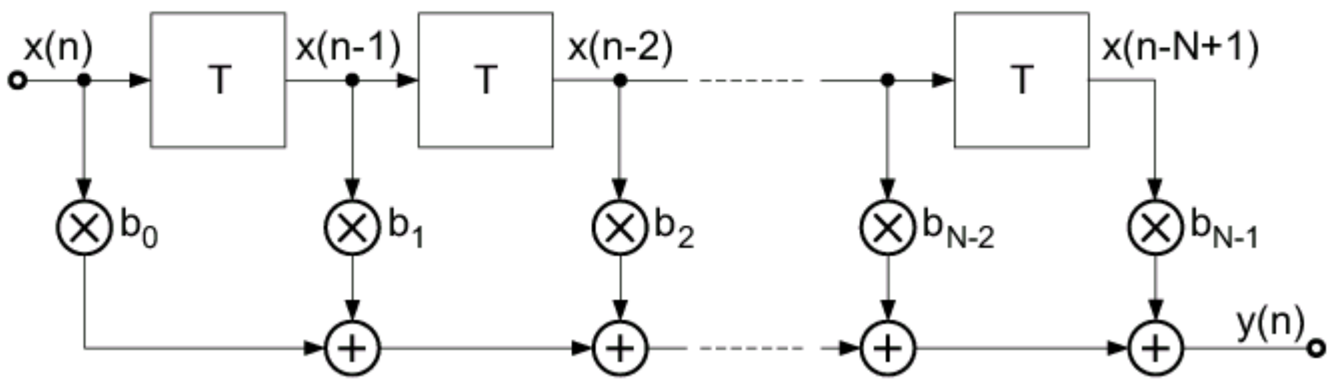
NONRECURSIVE EQUALIZERS

OUTLINE

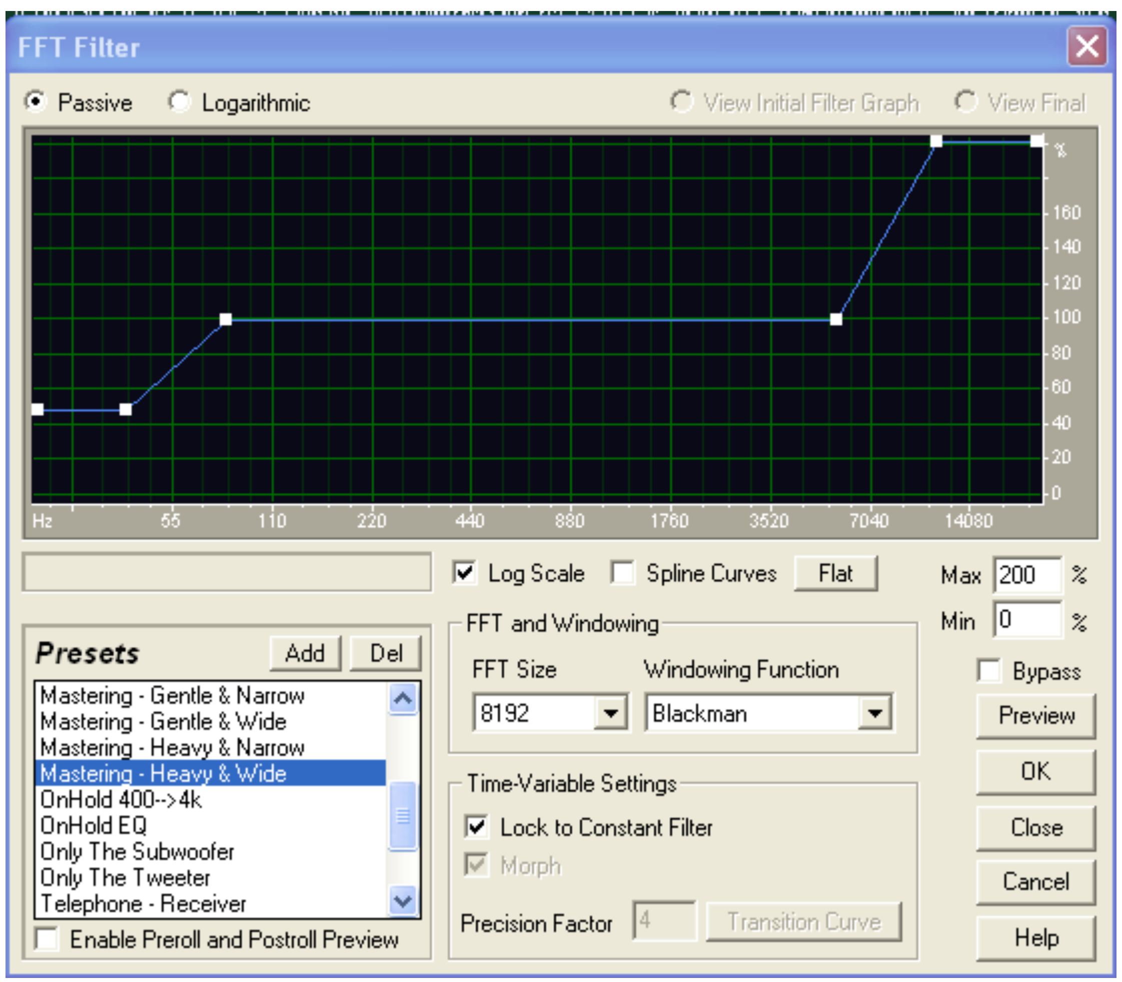
- Fast Convolution
- Fast Convolution of Long Sequences
- Filter Design by Frequency Sampling

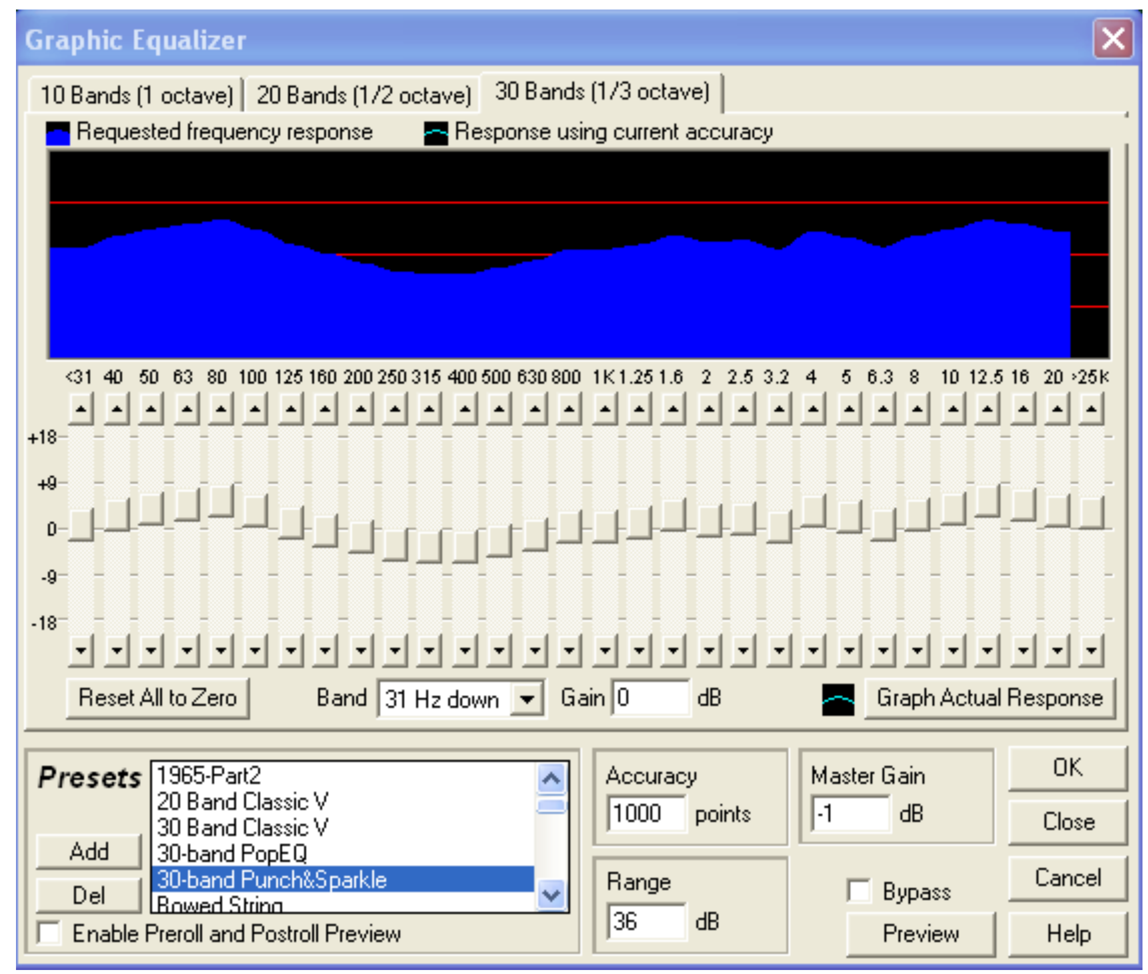


FIR: N Mul/sample

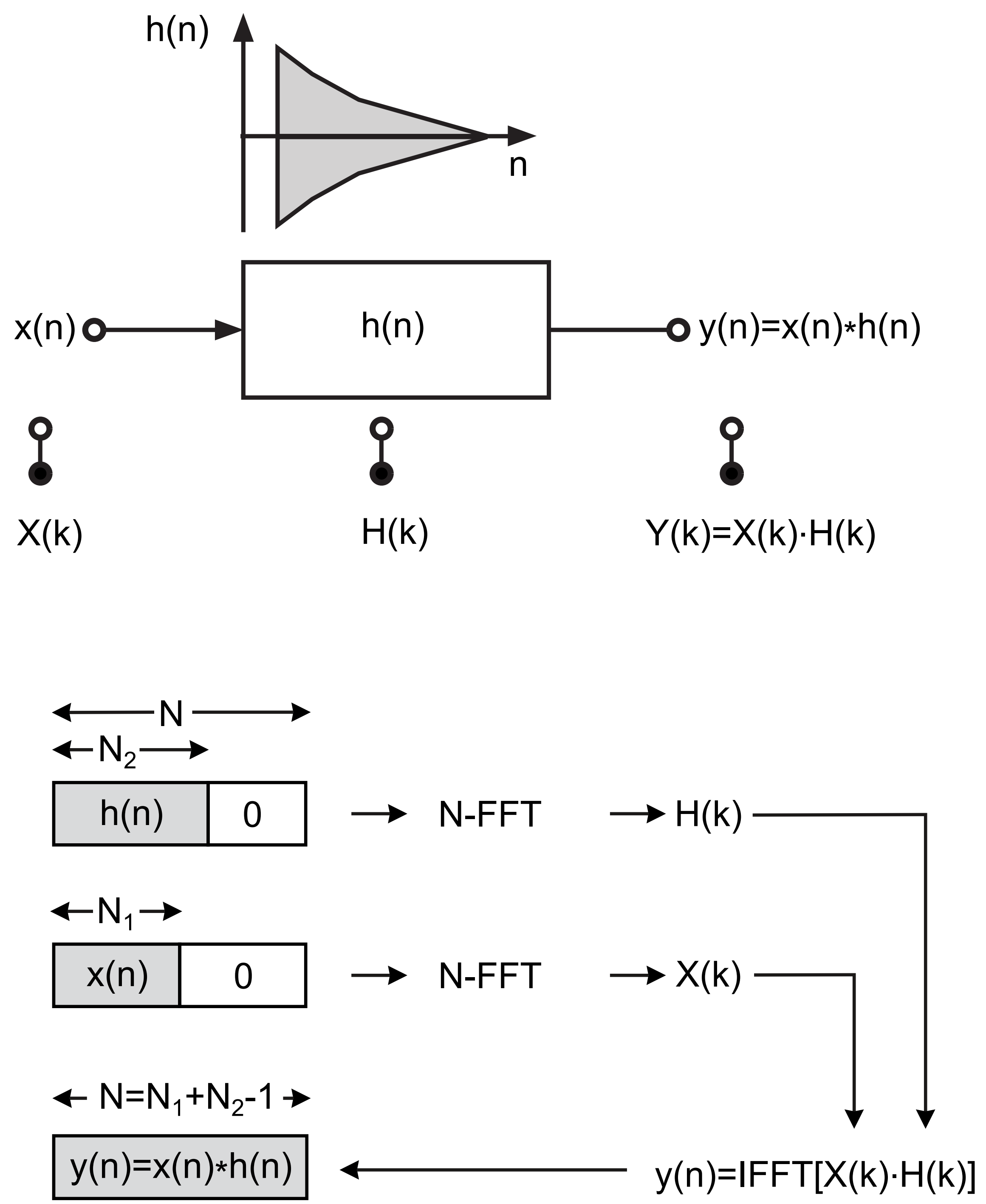
Fast Convolution: (2*2N*ld(2N)+2N)/2N=2*ld(2N)+1 Mul/sample

CONTROL INTERFACE AND PARAMETERS





FAST CONVOLUTION



DFT AND IDFT

DFT Algorithm

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \mathrm{DFT}_k[x(n)]$$

$$W_N = e^{-j2\pi/N}$$

IDFT Algorithm

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-nk}$$

Without scaling factor 1/N

$$x'(n) = \sum_{k=0}^{N-1} X(k) W_N^{-nk}$$

Symmetrical DFT/IDFT algorithms

$$X'(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X'(k) W_N^{-nk}$$

IDFT BY DFT

Exchange of real and imaginary part by conjugation and multiplication by j

$$\begin{aligned} x(n) &= a(n) + j \cdot b(n) \\ j \cdot x^*(n) &= b(n) + j \cdot a(n) \end{aligned}$$

Conjugation of IDFT

$$x'(n) = \sum_{k=0}^{N-1} X(k) W_N^{-nk} \qquad x'^*(n) = \sum_{k=0}^{N-1} X^*(k) W_N^{nk}$$

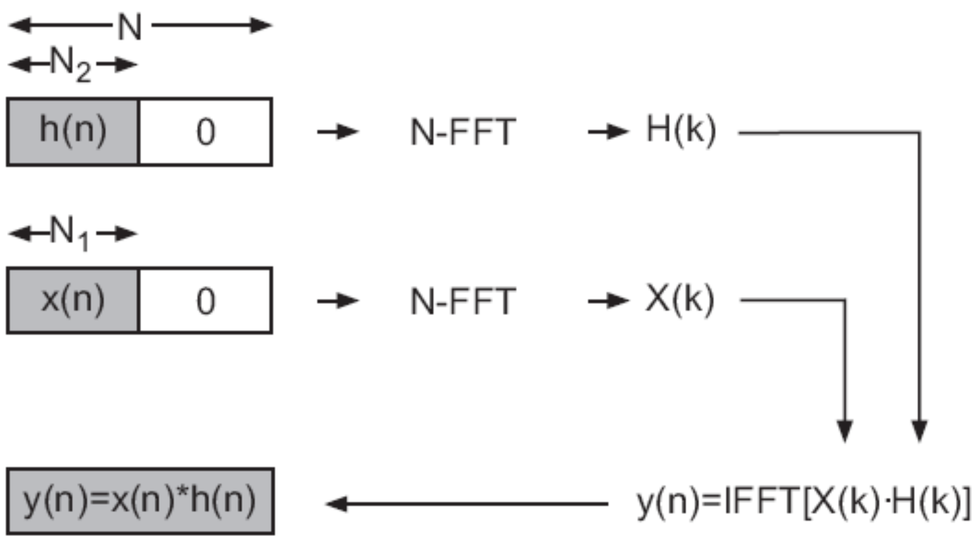
Multiplication by j

$$j \cdot x'^*(n) = \sum_{k=0}^{N-1} j \cdot X^*(k) W_N^{nk}$$

Conjugation and multiplication by j

$$x'(n) = j \cdot \left[\sum_{k=0}^{N-1} j \cdot X^*(k) W_N^{nk} \right]^*$$

FC WITH IDFT BY DFT



Fast Convolution
y(n)=x(n)*h(n) ○—○ Y(k)=X(k)H(k)

- Exchange real and imaginary part of Y(k)

$$Y(k) = Y_I(k) + jY_R(k)$$

- Perform DFT of $Y_I(k) + jY_R(k)$

$$\mathrm{DFT}[Y(k)] = y_I(n) + jy_R(n)$$

- Exchange real and imaginary part of x´(n)

$$y(n) = y_R(n) + jy_I(n)$$

PARTITIONING OF LONG INPUT SEQUENCES

$x_1(n)$	$x_2(n)$	$x_3(n)$

$$x_m(n) = \begin{cases} x(n) & (m-1)L \le n \le mL-1 \\ 0 & \text{sonst} \end{cases}$$

$$x(n) = \sum_{m=1}^{\infty} x_m(n)$$

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

$$= \sum_{k=0}^{M-1} h(k) \sum_{m=1}^{\infty} x_m(n-k)$$

Convolution sum with short input sequence $x_m(n)$

$$= \sum_{m=1}^{\infty} \left[\sum_{k=0}^{M-1} h(k)x_m(n-k) \right]$$

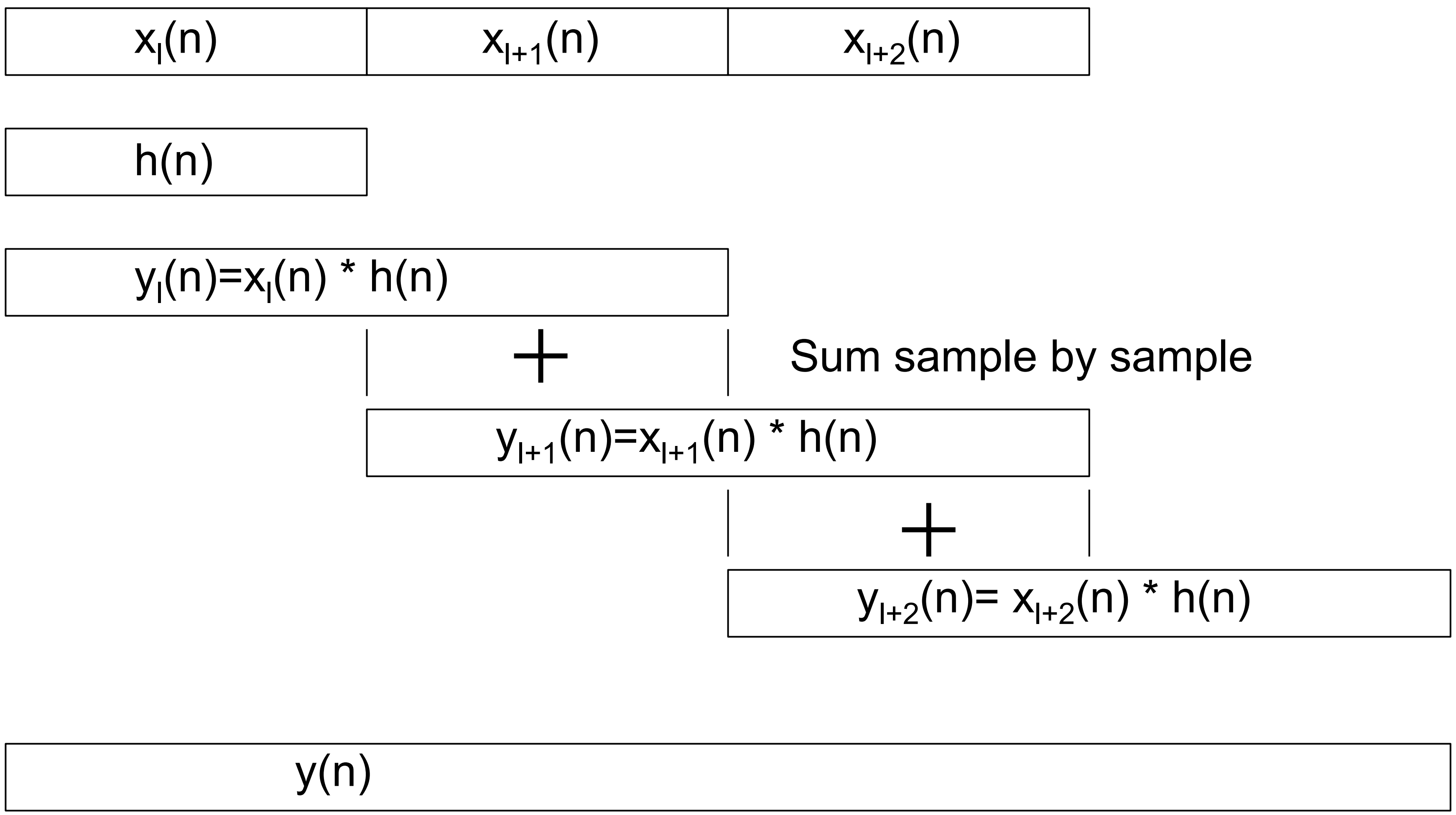
Sum over partioned convolution sums $y_m(n)$

$$y_m(n) = \begin{cases} \sum_{k=0}^{M-1} h(k)x_m(n-k) & (m-1)L \le n \le mL+M-2 \\ 0 & \text{sonst} \end{cases}$$

$$y(n) = \sum_{m=1}^{\infty} y_m(n)$$

Sum all convolution sums $y_m(n)$

OVERLAP AND ADD



PARTITIONING OF LONG IMPULSE RESPONSES

Partitioning of impulse response of length M into P smaller parts

$$h_p(n-(p-1)\frac{M}{P}) \quad = \quad \begin{cases} h(n) & (p-1)\frac{M}{P} \leq n \leq p\frac{M}{P}-1 \\ 0 & \text{sonst} \end{cases}$$

$$h(n) = \sum_{p=1}^{P} h_p(n-(p-1)\frac{M}{P})$$

$h_1(n)$	$h_2(n)$	$h_3(n)$	$h_4(n)$

$$y(n) \quad = \quad \sum_{m=1}^{\infty} \underbrace{[\sum_{k=0}^{M-1} h(k) x_m(n-k)]}_{y_m(n)}$$

$$= \quad \sum_{m=1}^{\infty} [\sum_{k=0}^{M_1-1} h(k) x_m(n-k) + \sum_{k=M_1}^{M_2-1} h(k) x_m(n-k) + \ldots$$

$$+ \sum_{k=M_{P-1}}^{M-1} h(k) x_m(n-k)] \quad .$$

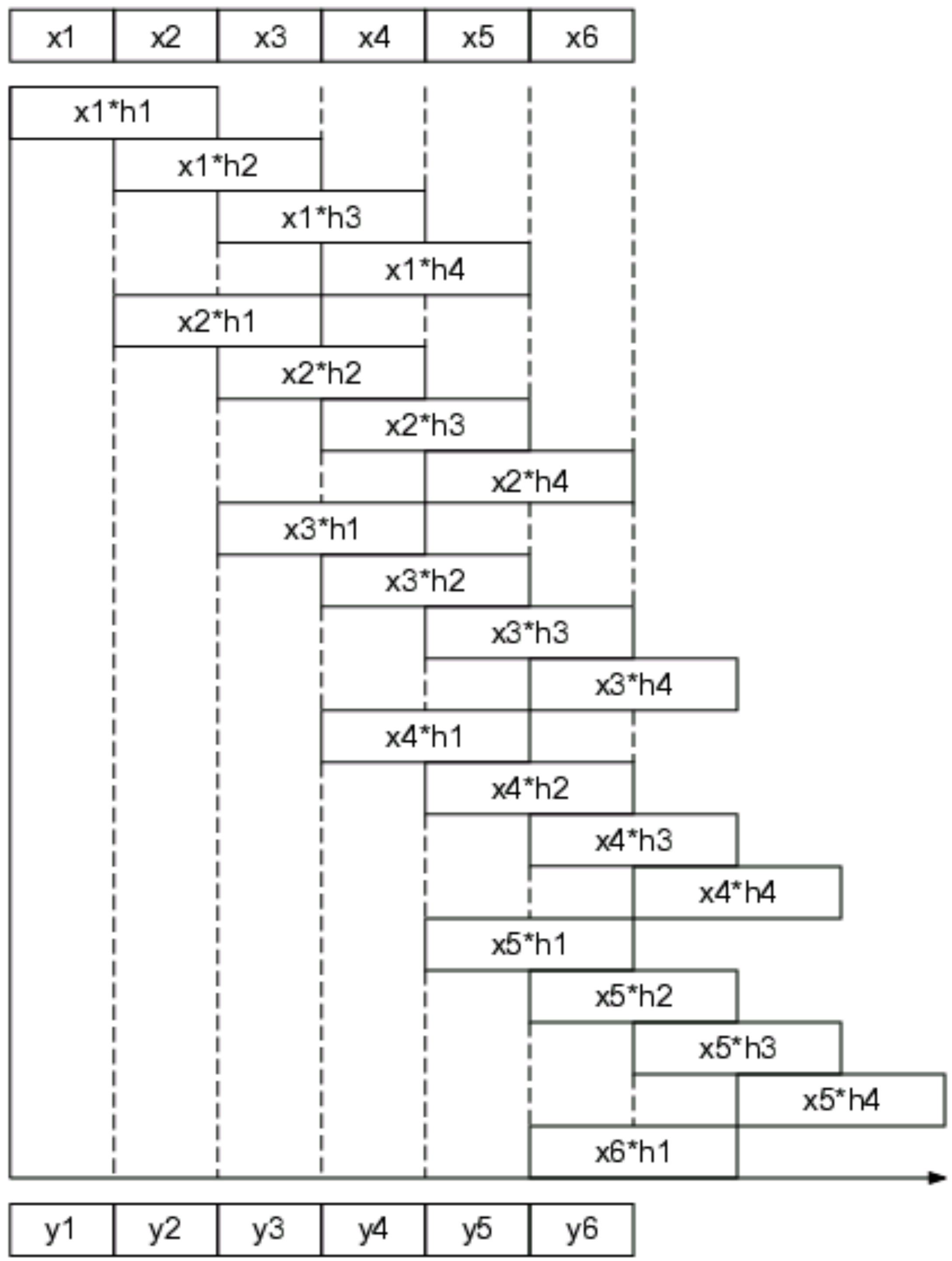
PARTITIONING OF LONG IMPULSE RESPONSES + OLA

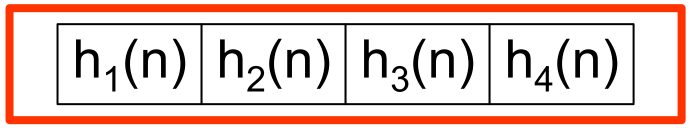
$$\begin{aligned}
y(n) &= \sum_{m=1}^{\infty} [\underbrace{\sum_{k=0}^{M_1-1} h_1(k) x_m(n-k)}_{y_{m1}} + \underbrace{\sum_{k=0}^{M_1-1} h_2(k) x_m(n-M_1-k)}_{y_{m2}} \\
&\quad + \underbrace{\sum_{k=0}^{M_1-1} h_3(k) x_m(n-2M_1-k)}_{y_{m3}} \\
&\quad \ldots + \underbrace{\sum_{k=0}^{M_1-1} h_P(k) x_m(n-(P-1)M_1-k)}_{y_{mP}}] \\
&= \sum_{m=1}^{\infty} \underbrace{[y_{m1}(n) + y_{m2}(n-M_1) + \ldots + y_{mP}(n-(P-1)M_1)]}_{y_m(n)}
\end{aligned}$$

PARTITIONING OF LONG IMPULSE RESPONSES P=4

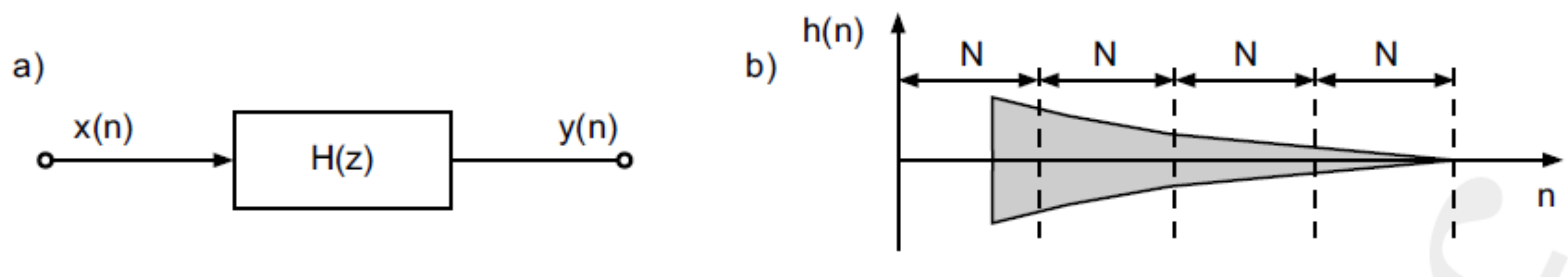
$$
\begin{aligned}
y(n) &= \sum_{m=1}^{\infty}[\underbrace{\sum_{k=0}^{M_1-1} h_1(k)x_m(n-k)}_{y_{m1}} + \underbrace{\sum_{k=0}^{M_1-1} h_2(k)x_m(n-M_1-k)}_{y_{m2}} \\
&\quad + \underbrace{\sum_{k=0}^{M_1-1} h_3(k)x_m(n-2M_1-k)}_{y_{m3}} + \underbrace{\sum_{k=0}^{M_1-1} h_4(k)x_m(n-3M_1-k)}_{y_{m4}}] \\
&= \sum_{m=1}^{\infty}[\underbrace{y_{m1}(n) + y_{m2}(n-M_1) + y_{m3}(n-2M_1) + y_{m4}(n-3M_1)}_{y_m(n)}]
\end{aligned}
$$

PARTITIONING OF LONG IMPULSE RESPONSES P=4

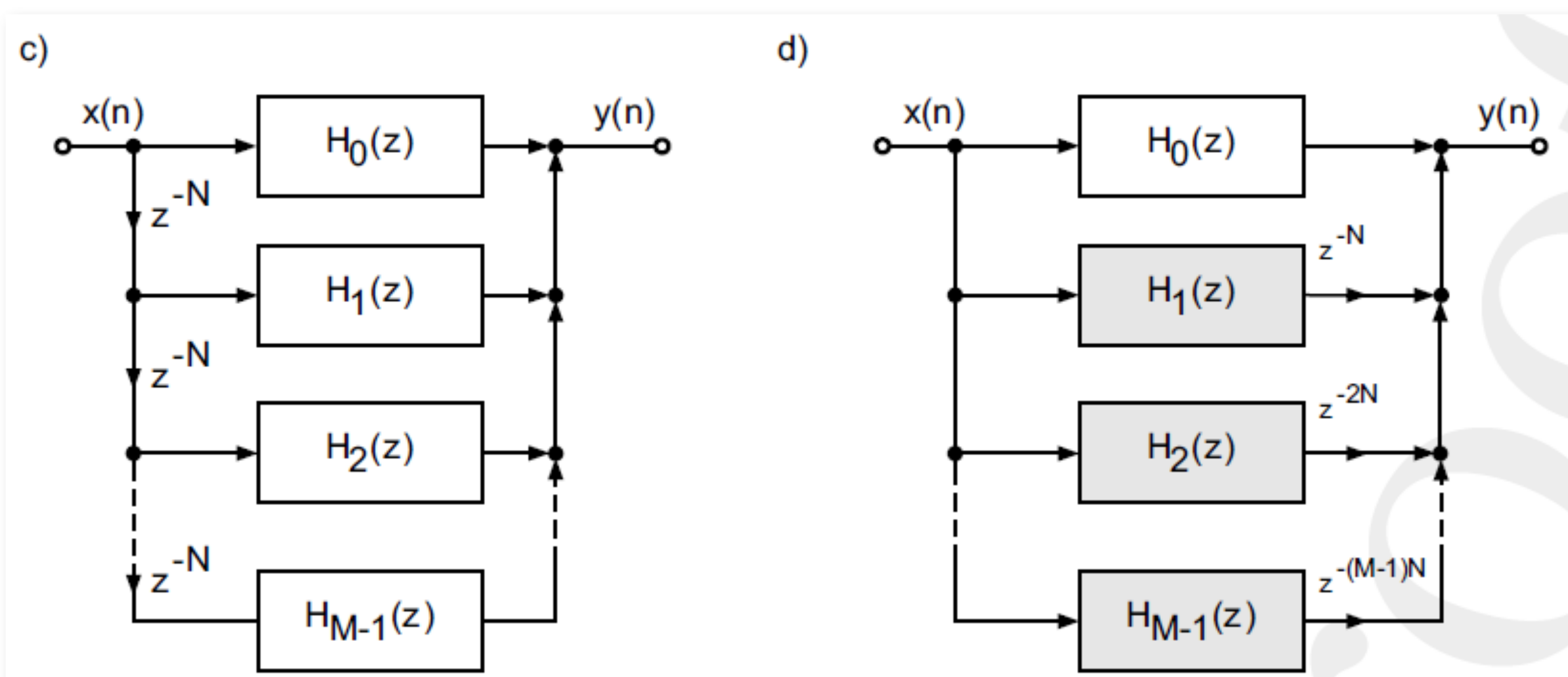




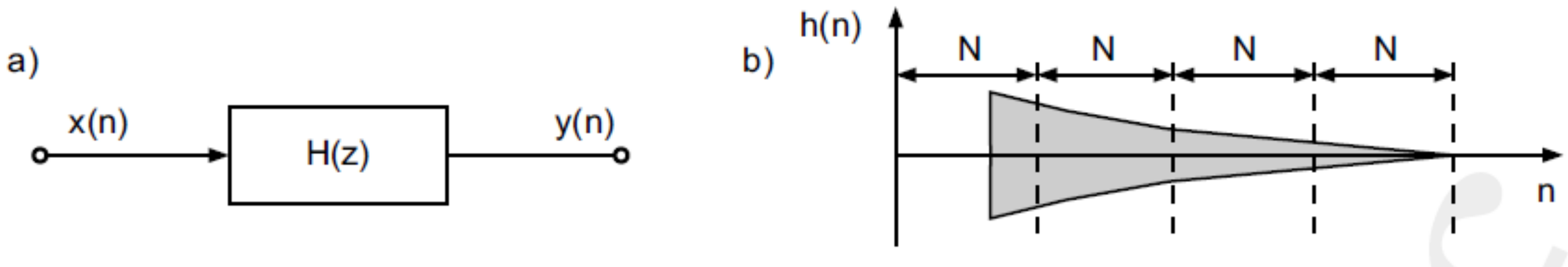
EFFICIENT FAST CONVOLUTION



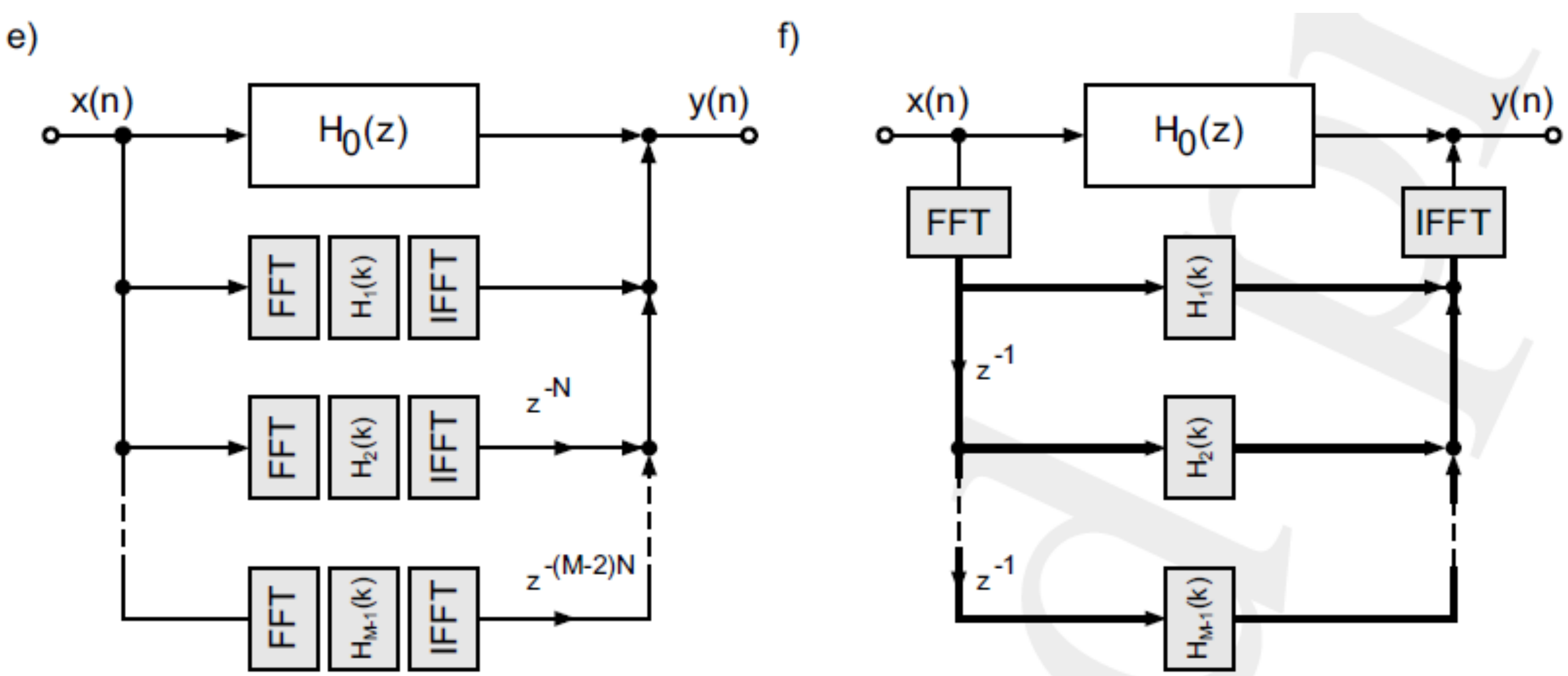
$$H(z) = \sum_{i=0}^{M-1} z^{-iN} H_i(z),$$



EFFICIENT FAST CONVOLUTION

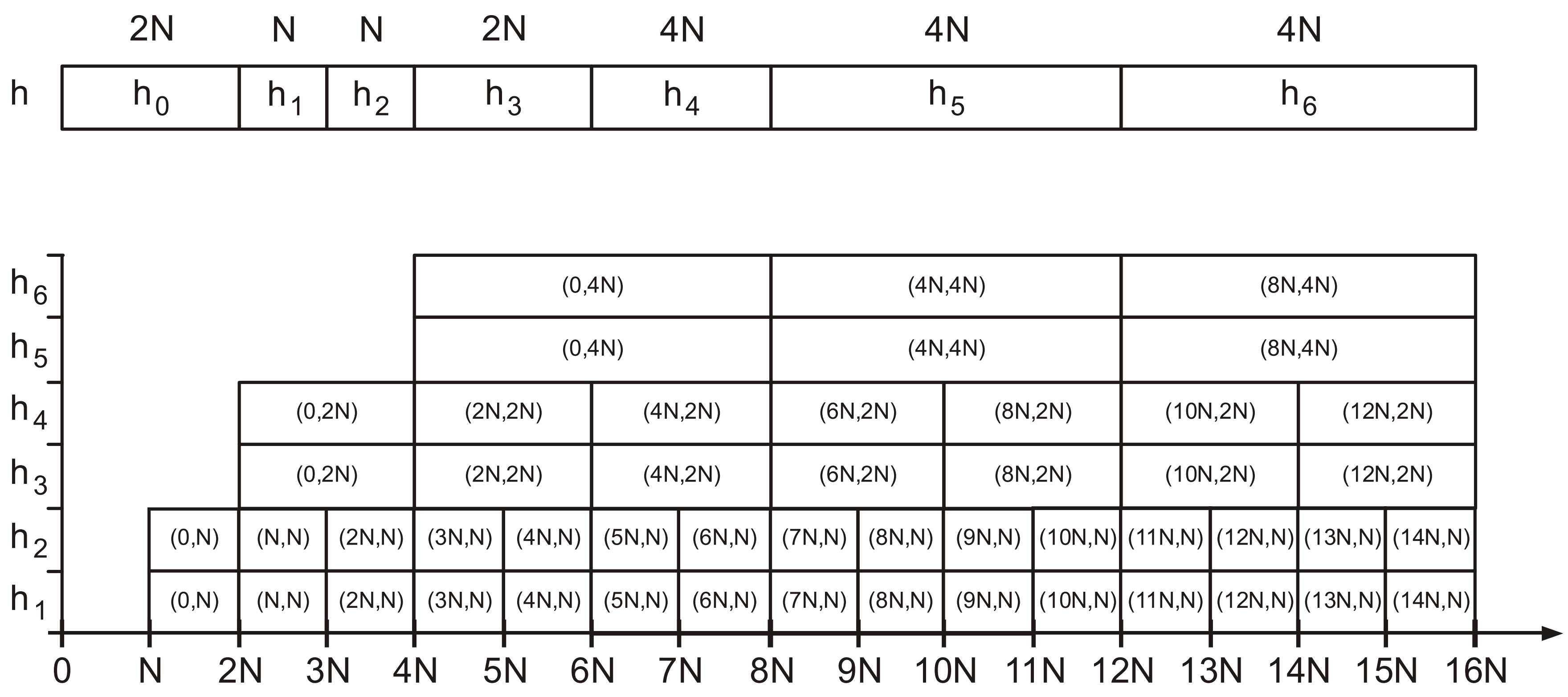


$$H(z) = \sum_{i=0}^{M-1} z^{-iN} H_i(z),$$



ZERO-DELAY FAST CONVOLUTION

Time domain convolution for h_0 and
fast convolution for $h_{1..6}$



FILTER DESIGN BY FREQUENCY SAMPLING

$$H(e^{j\Omega}) = |H(e^{j\Omega})|e^{j\varphi(\Omega)}, \text{ with } \varphi(\Omega) = -k\cdot\Omega$$

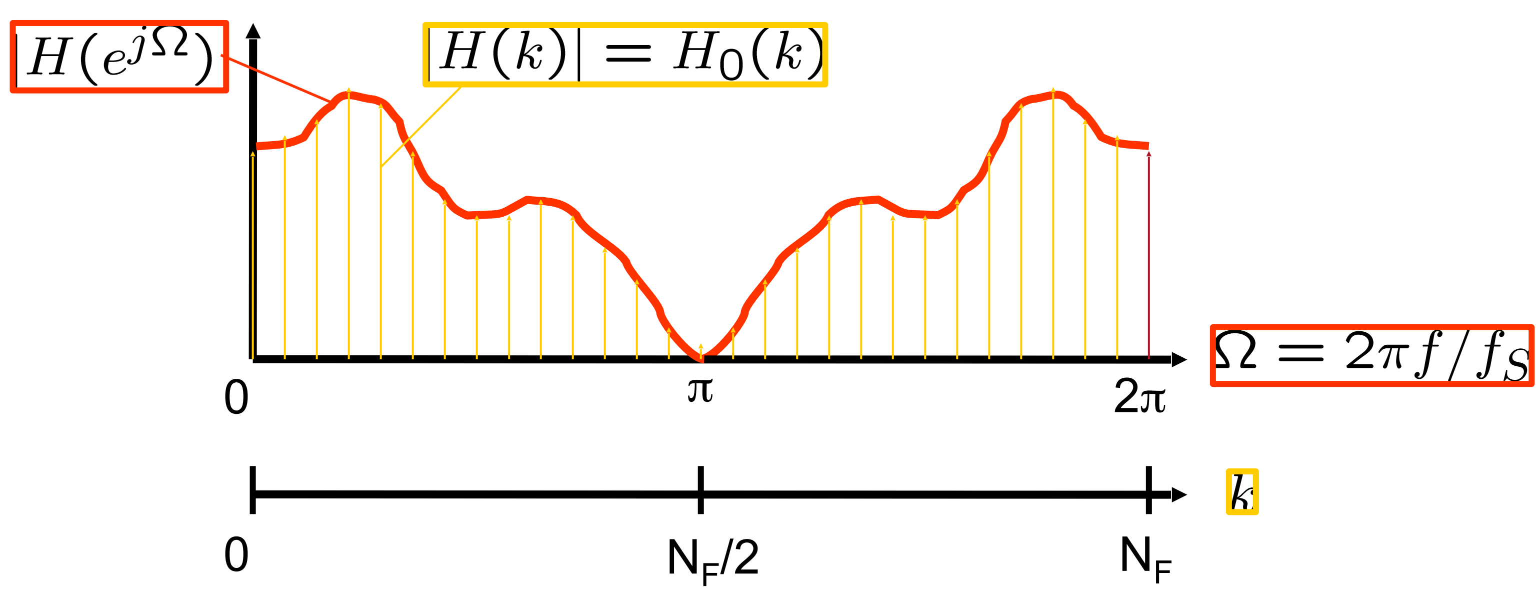
Frequency Response of Linear Phase Filter

$$H(e^{j\Omega}) = H_0(e^{j\Omega})e^{-j\frac{N_F-1}{2}\Omega}$$

N_F filter length

Frequency Sampling of the Magnitude

$$|H(e^{j\Omega})| = H_0(e^{j2\pi k/N_F}) \qquad k = 0, 1, ..., \frac{N_F}{2} - 1$$



FILTER DESIGN BY FREQUENCY SAMPLING

$$H(e^{j\Omega}) = H_0(e^{j\Omega})e^{-j\frac{N_F-1}{2}\Omega} \qquad \frac{f}{f_A} = \frac{k}{N_F} \qquad \text{with} \qquad k = 0, 1, ..., N_F - 1$$

Frequency Sampling of the Magnitude

$$|H(e^{j\Omega})| = H_0(e^{j2\pi k/N_F}) \qquad k = 0, 1, ..., \frac{N_F}{2} - 1$$

Linear Phase Constraint

$$e^{-j\frac{N_F-1}{2}\Omega} = e^{-j2\pi\frac{N_F-1}{2}\frac{k}{N_F}}$$

N_F filter length

$$= \cos(2\pi\frac{N_F-1}{2}\frac{k}{N_F}) - j\sin(2\pi\frac{N_F-1}{2}\frac{k}{N_F})$$

$$k = 0, 1, ..., \frac{N_F}{2} - 1.$$

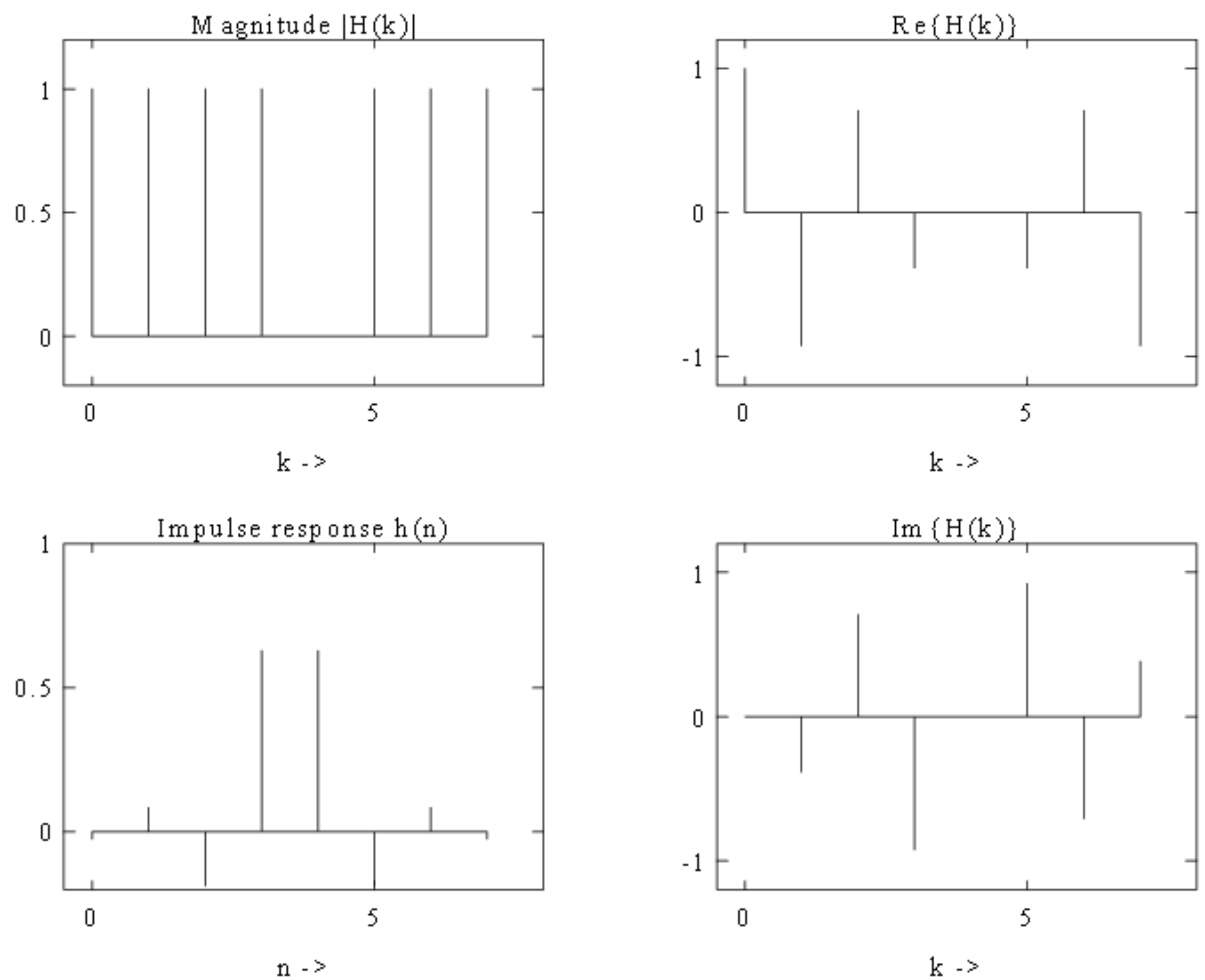
Real Impulse Response

$$H(k) = H^*(N_F - k) \qquad k = 0, 1, ... \frac{N_F}{2} - 1$$

$$H(k = \frac{N_F}{2}) = 0$$

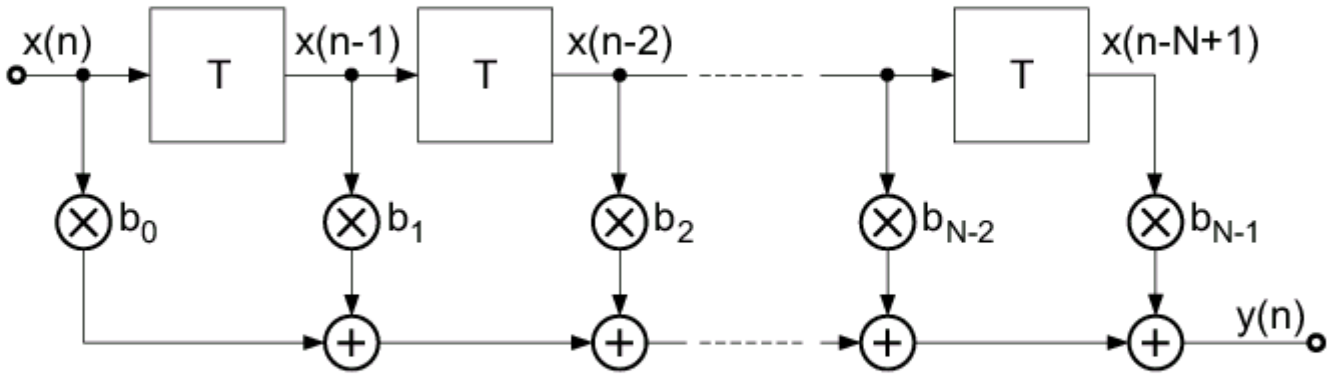
Condition for even N length impulse response

FREQUENCY SAMPLING – EXAMPLE



$$h(n) = \mathrm{IDFT}\{H(k)\} = \mathrm{IDFT}\{H_R(k) + jH_I(k)\}$$

COMPUTATIONAL COMPLEXITY



FIR: N Mul/sample
Fast Convolution: (2*2N*ld(2N)+2N)/2N=2*ld(2N)+1 Mul/sample

FIR (N=1024): 1024 Mul/sample
Fast Convolution (N=1024): 23 Mul/sample