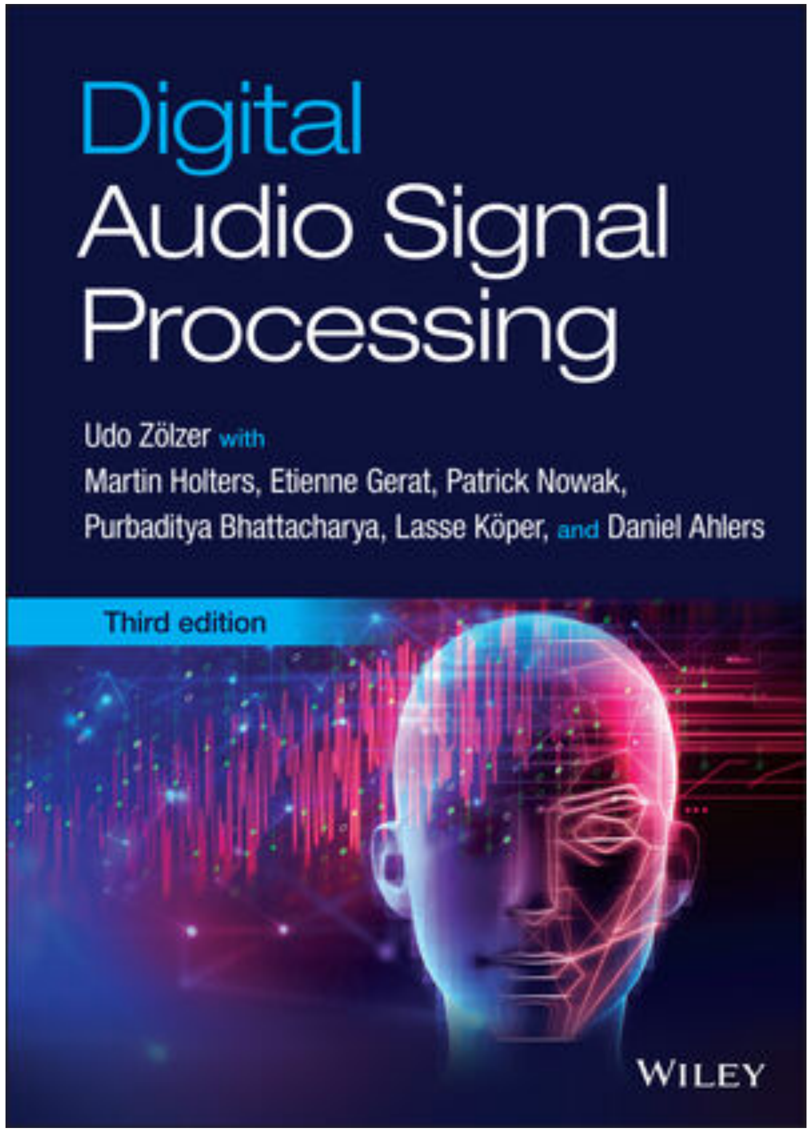


Lecture by Udo Zölzer

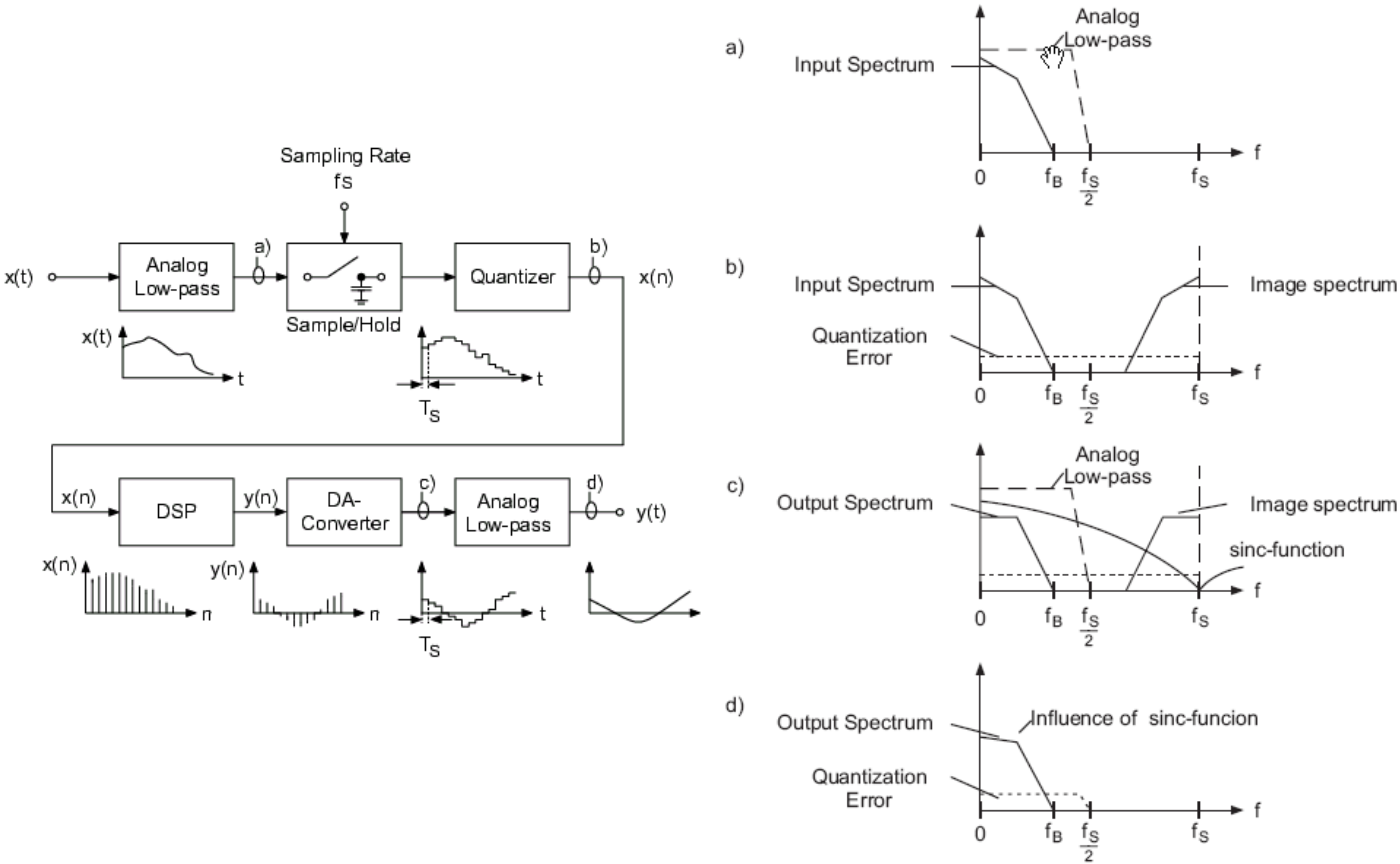
- Introduction
- Quantization
- Sampling Rate Conversion
- AD/DA Conversion
- Equalizers
- Room Simulation
- Dynamic Range Control
- Audio Coding
- Nonlinear Processing
- Machine Learning for Audio

AD/DA CONVERSION

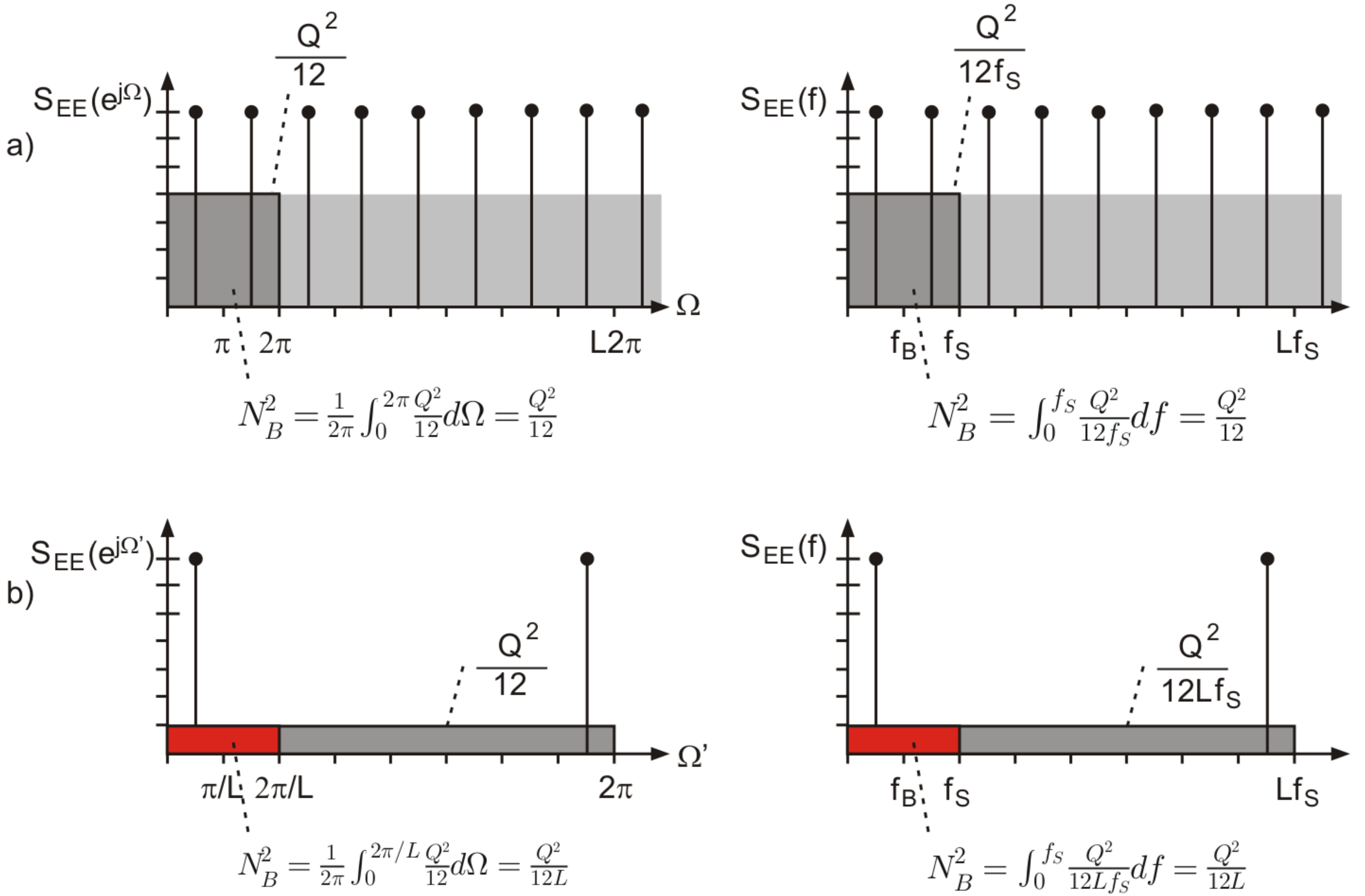
OUTLINE

- Nyquist Sampling
- Oversampling Techniques
- Delta-sigma Conversion

NYQUIST SAMPLING

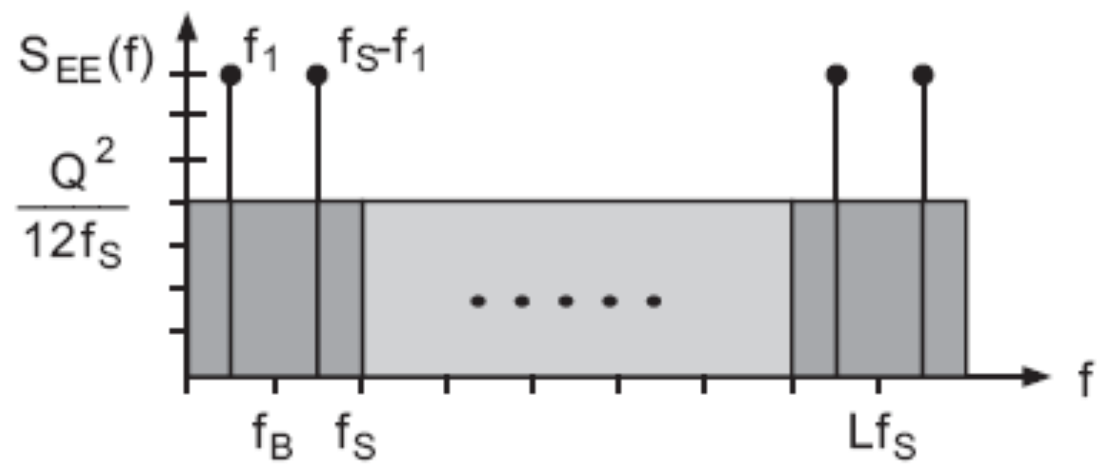


OVERSAMPLING TECHNIQUES

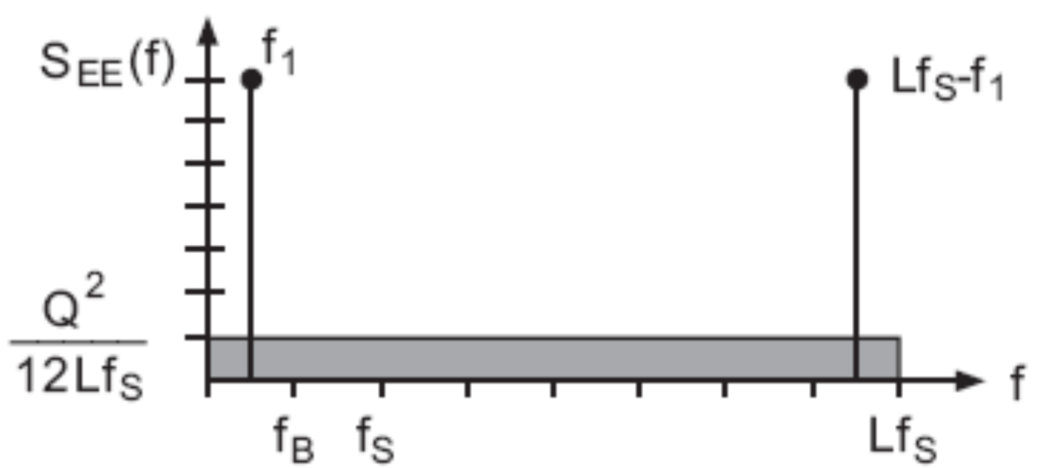


OVERSAMPLING TECHNIQUES

a) Nyquist sampling



b) Oversampling



PDS of quantization error

$$S_{EE}(f) = \frac{Q^2}{12 f_S}$$

Noise power in audio band

$$N_B^2 = \sigma_E^2 = 2\int_0^{f_B} S_{EE}(f)df = \frac{Q^2}{12}.$$

PDS of quantization error

$$S_{EE}(f) = \frac{Q^2}{12 L f_S}$$

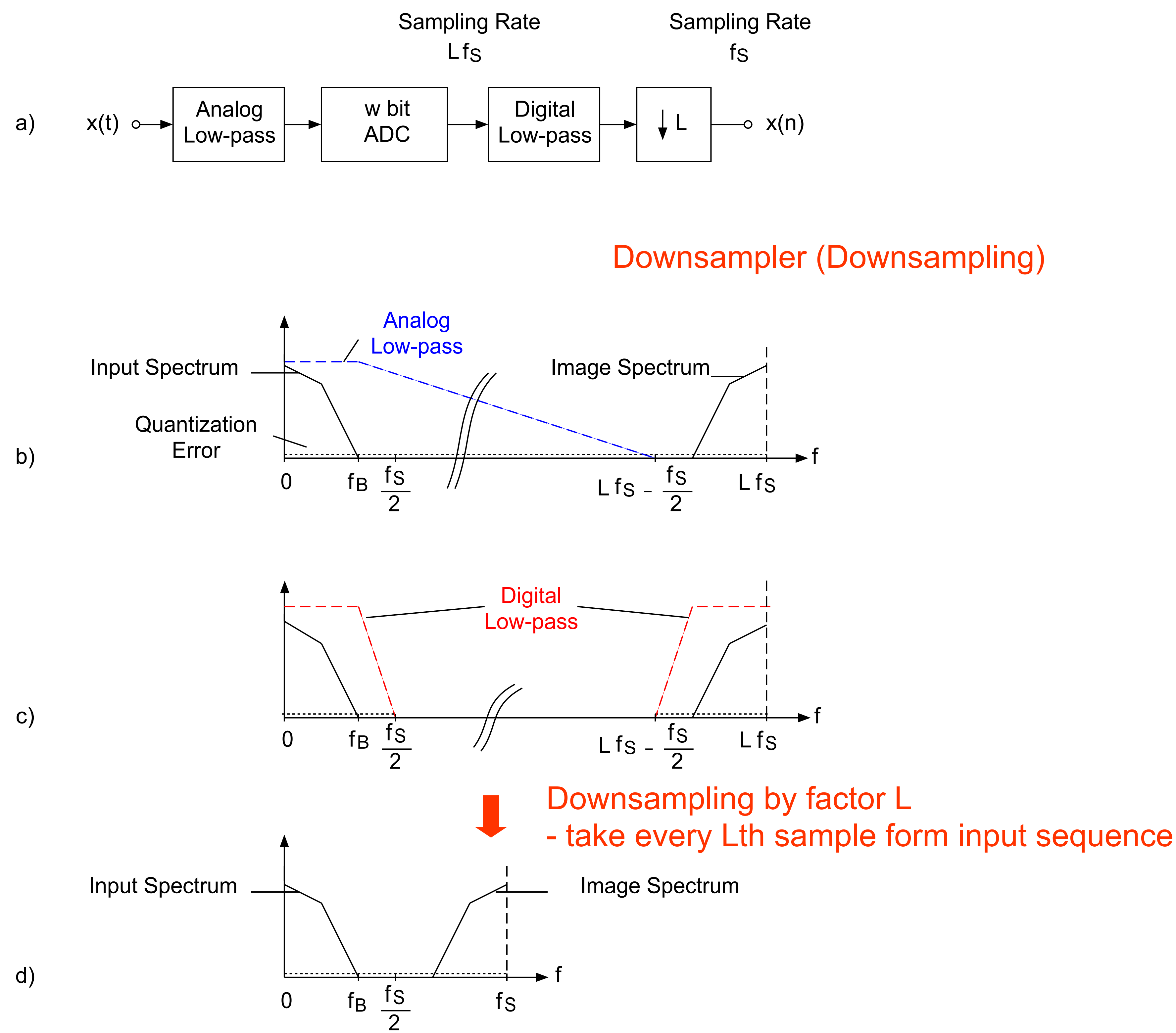
Noise power in audio band

$$N_B^2 = 2 f_B \frac{Q^2}{12 L f_S} = \frac{Q^2}{12}\frac{1}{L}.$$

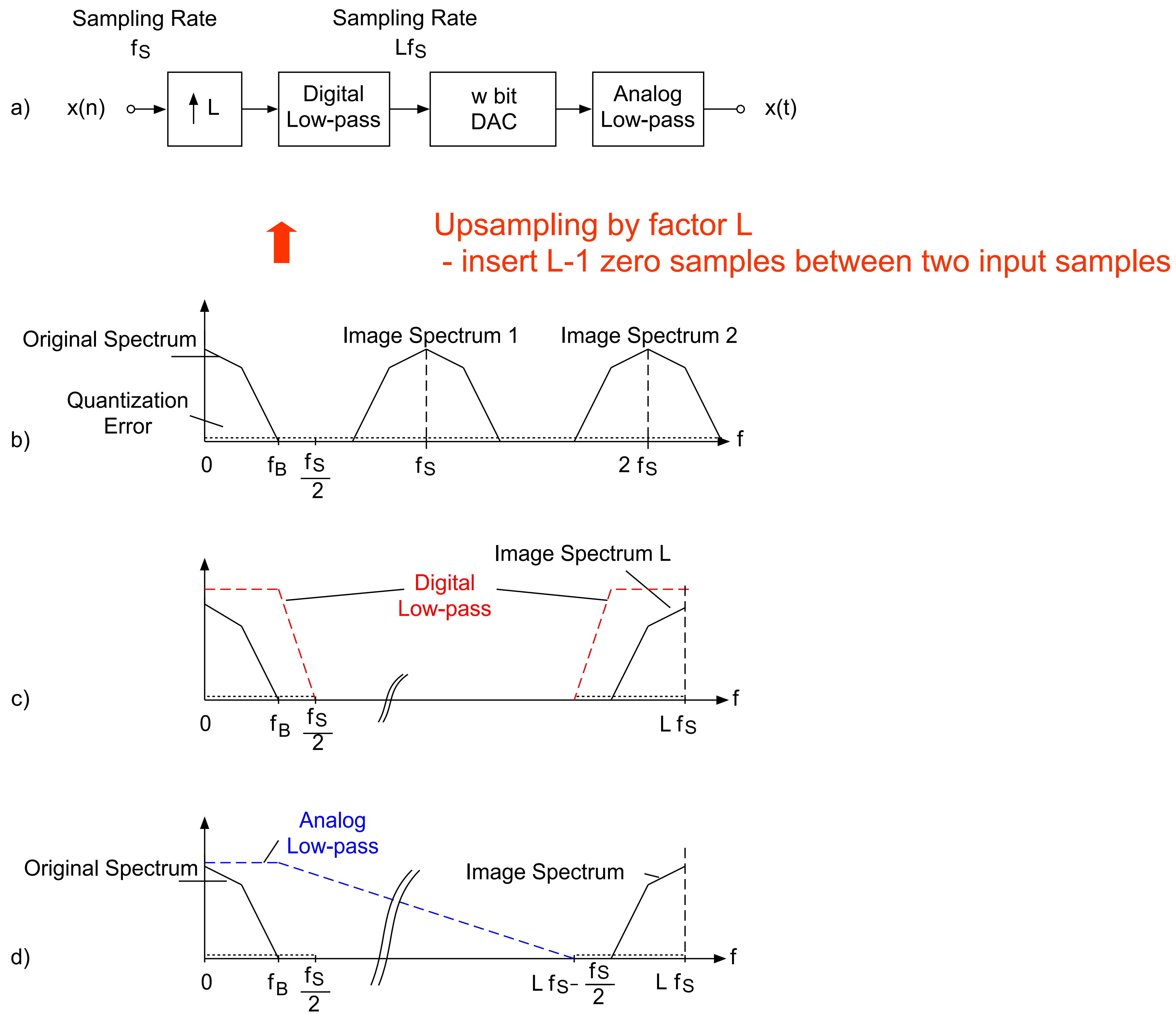
SNR with oversampling

$$\mathrm{SNR} = 6.02 \cdot w + 10\log_{10}(L) \qquad \mathrm{dB}.$$

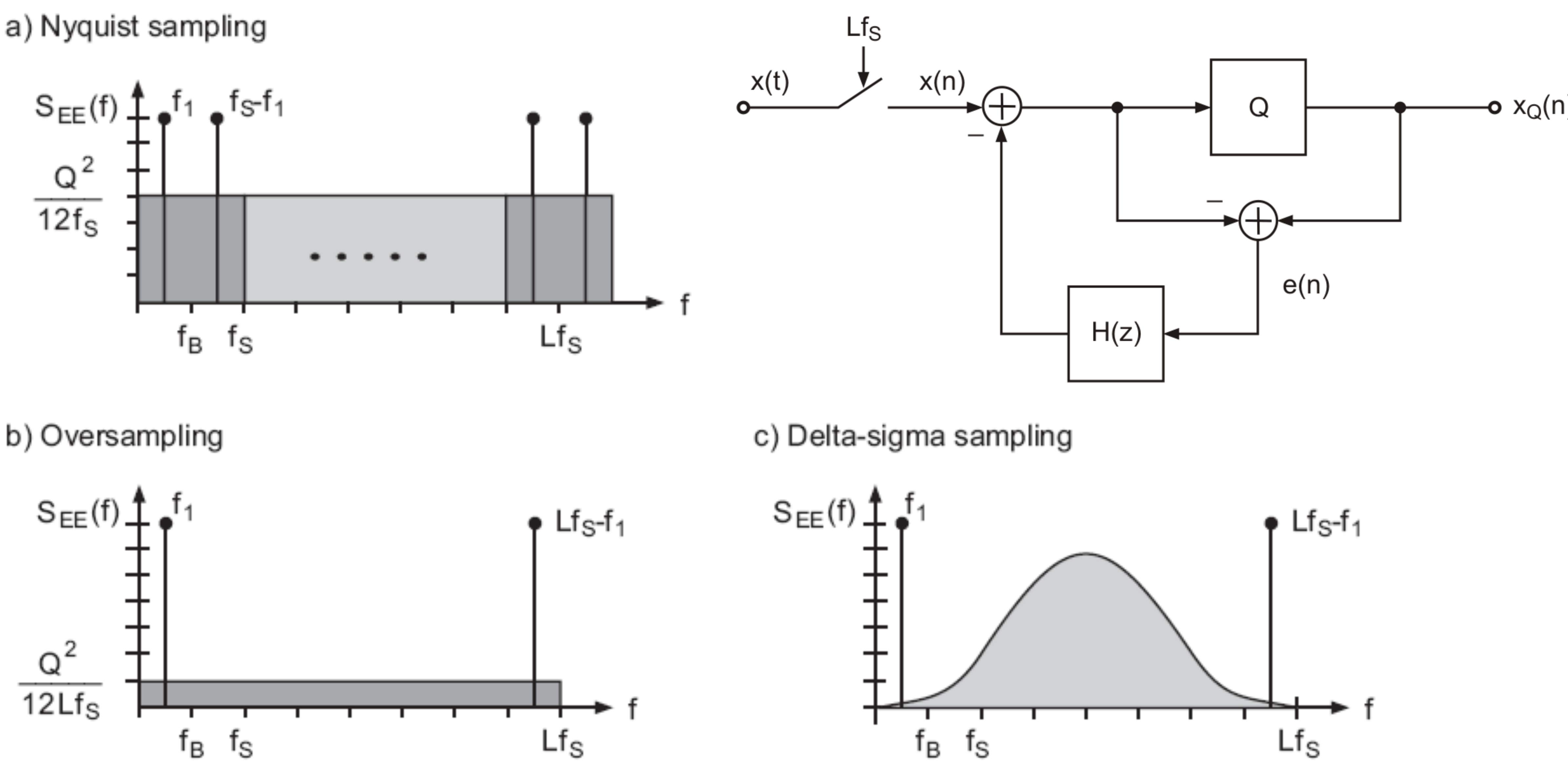
OVERSAMPLING AD CONVERTER



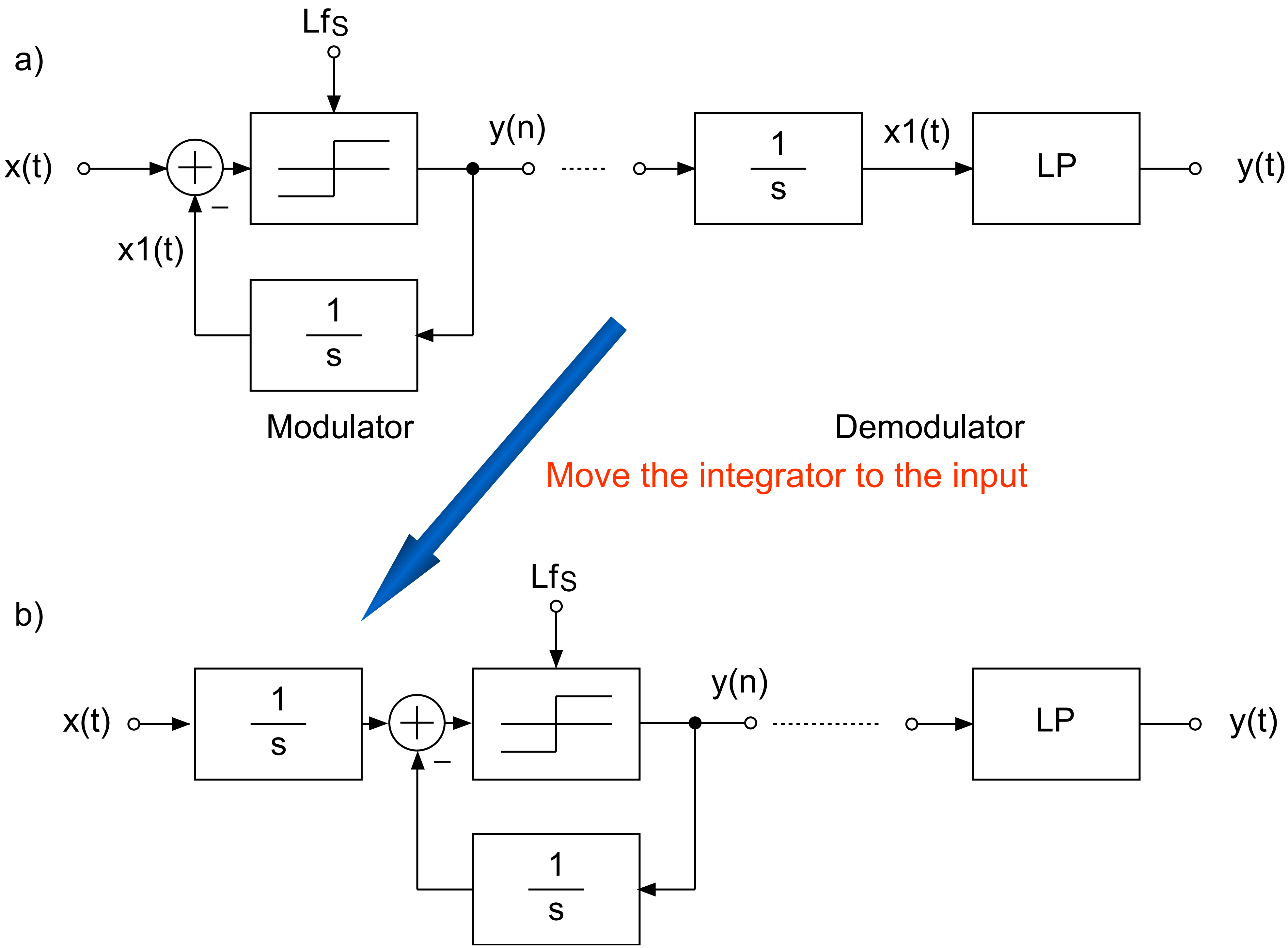
OVERSAMPLING DA CONVERTER



DELTA-SIGMA CONVERSION

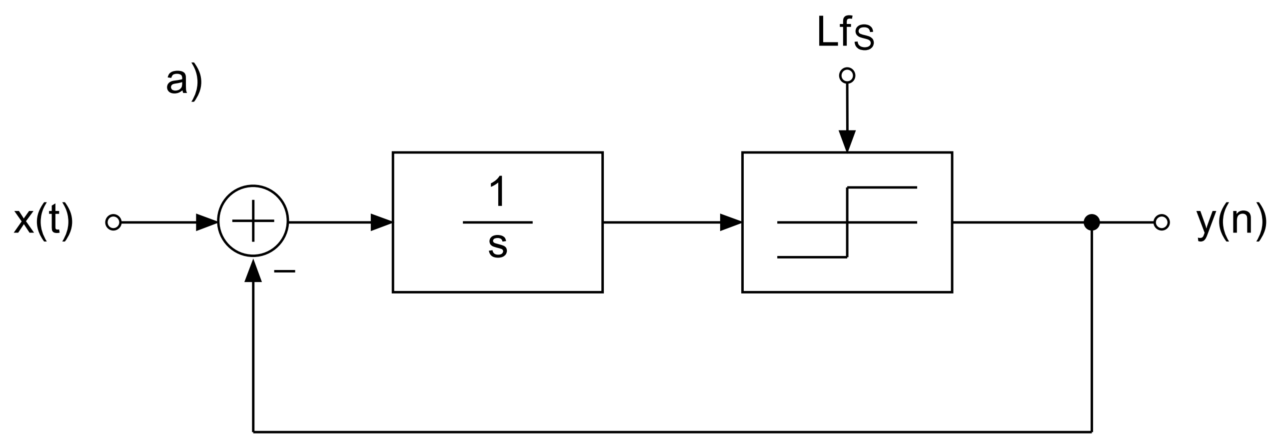


DELTA MODULATION

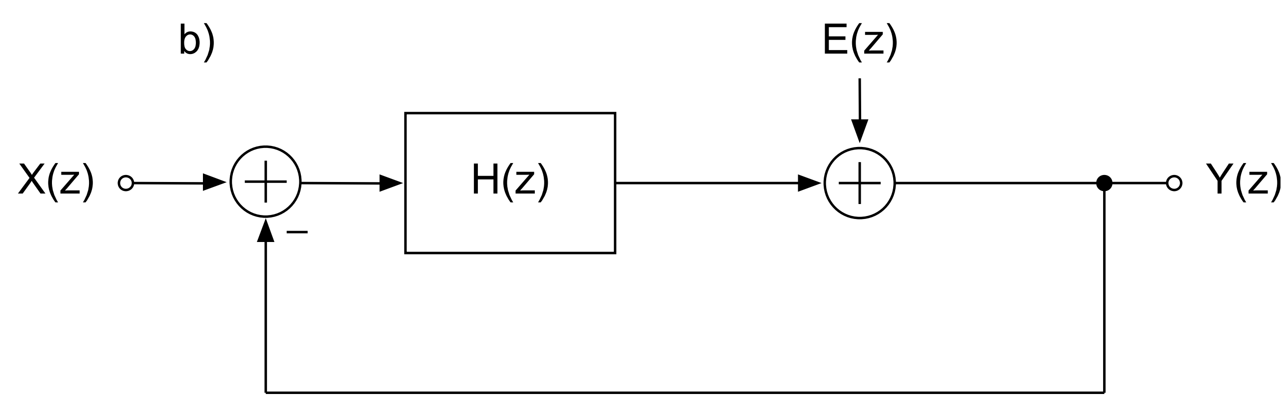


DELTA-SIGMA MODULATION

Continuous-time delta-sigma modulator



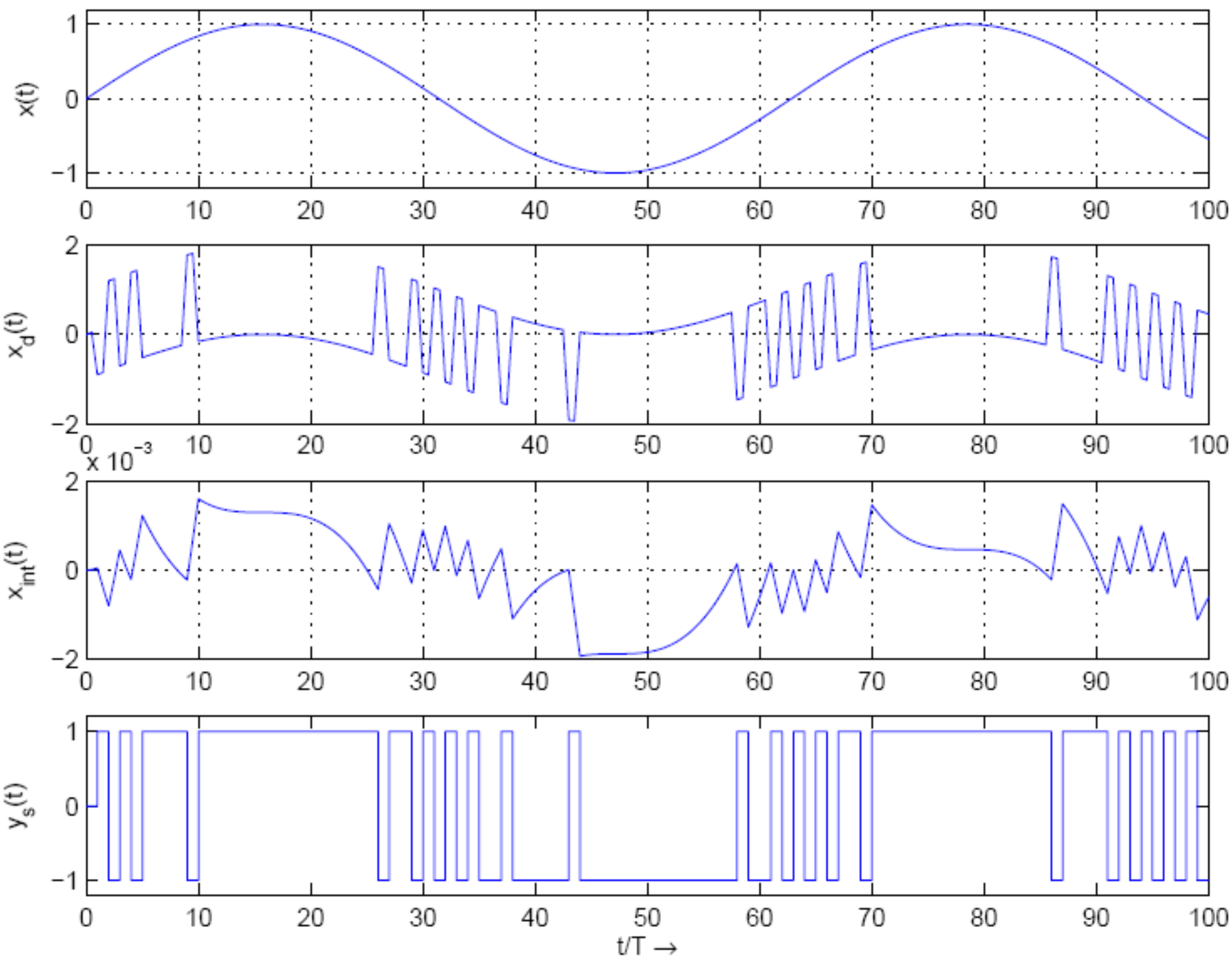
Discrete-time delta-sigma modulator



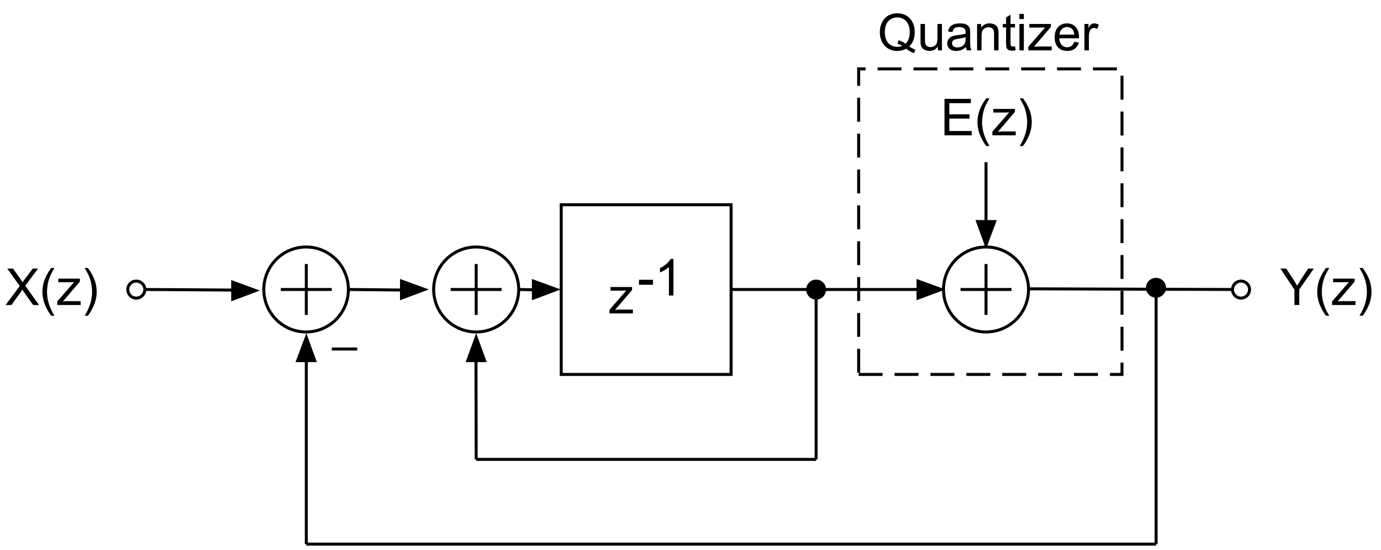
Z-transform of output signal

$$
\begin{aligned}
Y(z) &= \frac{H(z)}{1+H(z)}X(z) + \frac{1}{1+H(z)}E(z) \\
&\approx X(z) + \frac{1}{1+H(z)}E(z) \quad .
\end{aligned}
$$

DELTA-SIGMA MODULATION - SIGNALS



FIRST-ORDER DSM



Difference equation

$$y(n) = x(n-1) + e(n) - e(n-1).$$

Z-Transform

$$Y(z) = z^{-1}X(z) + E(z)\underbrace{\left(1 - z^{-1}\right)}_{H_E(z)}.$$

PDS of output error

$$S_{E_1E_1}(e^{j\Omega}) \quad = \quad S_{EE}(e^{j\Omega})\left|1 - e^{-j\Omega}\right|^2$$

FIRST-ORDER DSM

Output error

$$e_1(n) = e(n) - e(n-1)$$

PDS of output error

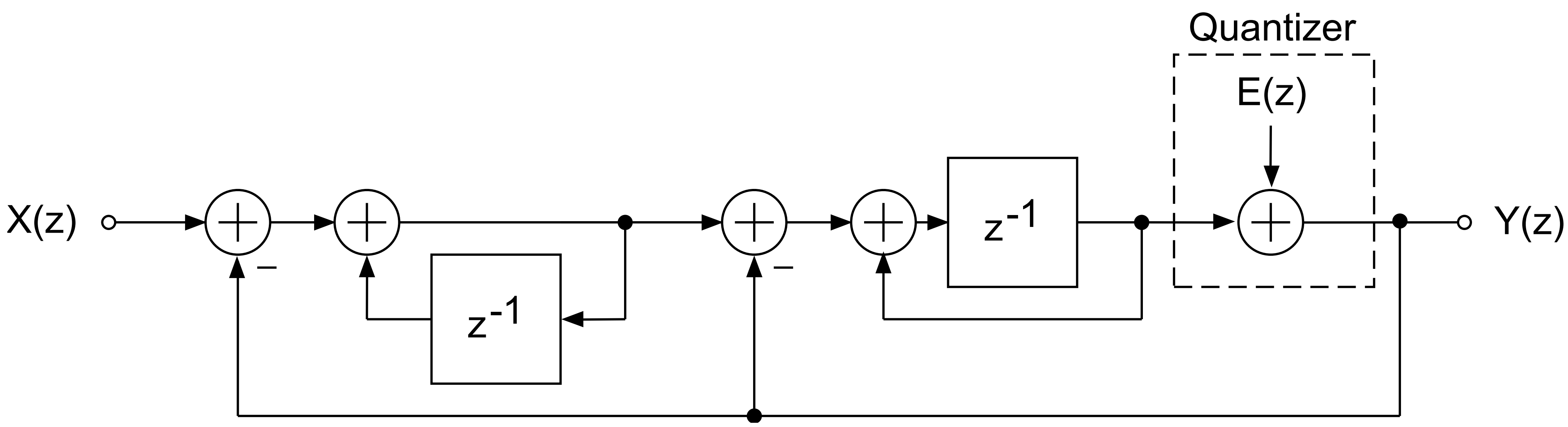
$$\begin{aligned} S_{E_1E_1}(e^{j\Omega}) &= S_{EE}(e^{j\Omega})\left|1 - e^{-j\Omega}\right|^2 \\ &= S_{EE}(e^{j\Omega})4\sin^2(\frac{\Omega}{2}), \qquad S_{EE}(f) = \tfrac{Q^2}{12Lf_S} \end{aligned}$$

Noise power in base band

$$\begin{aligned} N_B^2 &= S_{EE}(f)2\int_0^{f_B} 4\sin^2(\pi\frac{f}{Lf_S})df \\ &\simeq \frac{Q^2}{12}\frac{\pi^2}{3}\left(\frac{2f_B}{Lf_S}\right)^3. \end{aligned}$$

$$N_B^2 = \frac{Q^2}{12}\frac{\pi^2}{3}\left(\frac{1}{L}\right)^3.$$

SECOND-ORDER DSM



Difference equation $y(n) = x(n-1) + e(n) - 2e(n-1) + e(n-2)$

z-Transform $Y(z) = z^{-1}X(z) + E(z)\underbrace{\left(1 - 2z^{-1} + z^{-2}\right)}_{H_E(z)=(1-z^{-1})^2}.$

PDS of output error $S_{E_1E_1}(e^{j\Omega}) = S_{EE}(e^{j\Omega})\left|1 - e^{-j\Omega}\right|^4$

SECOND-ORDER DSM

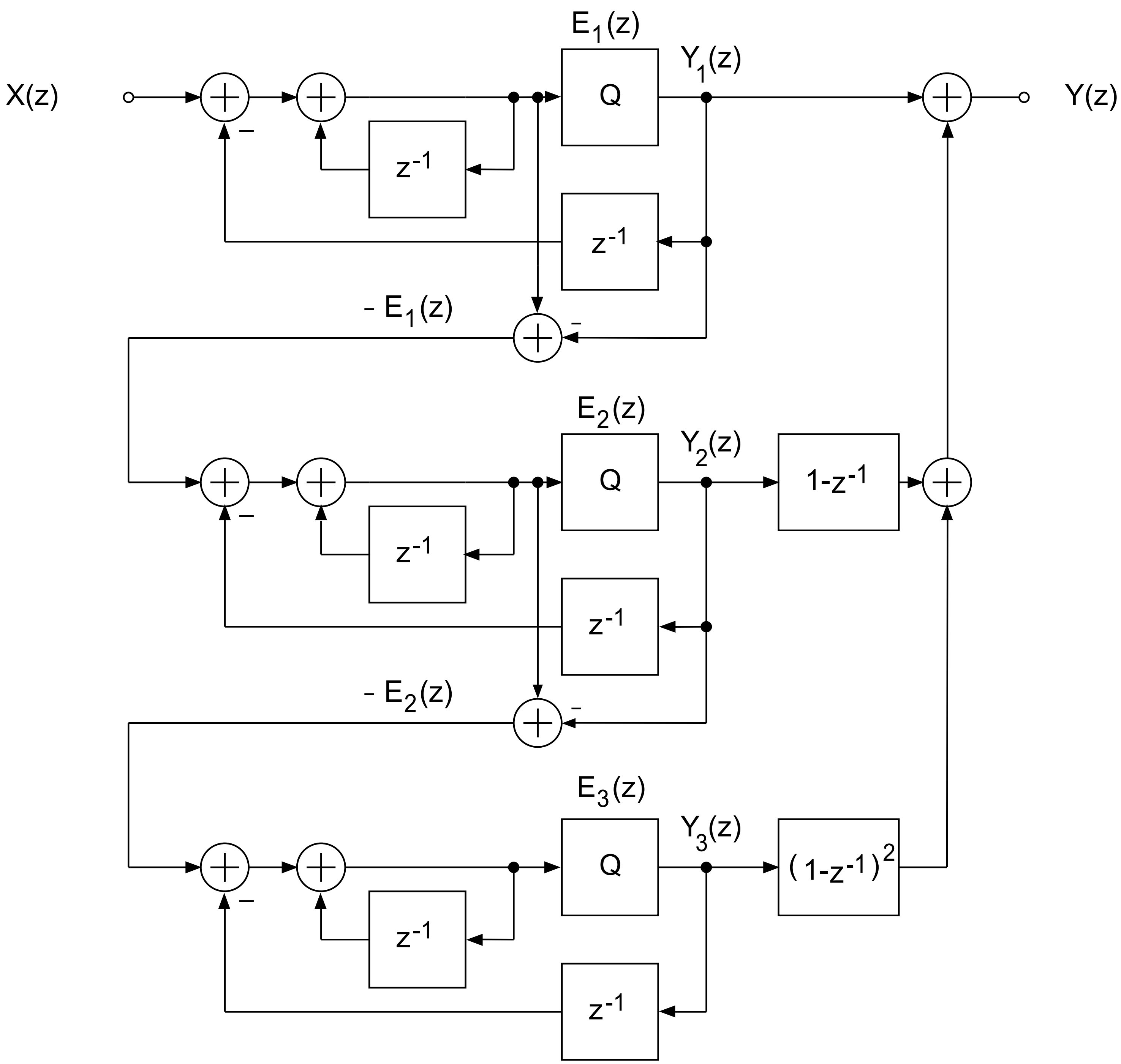
Output error $e_1(n) = e(n) - 2e(n-1) + e(n-2)$

PDS of output error
$$\begin{aligned} S_{E_1E_1}(e^{j\Omega}) &= S_{EE}(e^{j\Omega})\left|1 - e^{-j\Omega}\right|^4 \\ &= S_{EE}(e^{j\Omega})[4\sin^2(\frac{\Omega}{2})]^2 \\ &= S_{EE}(e^{j\Omega})4[1-\cos(\Omega)]^2. \end{aligned}$$

Noise power
in audio band
$$\begin{aligned} N_B^2 &= S_{EE}(f)2\int_0^{f_B} 4[1-\cos(\Omega)]^2 df \\ &\simeq \frac{Q^2}{12}\frac{\pi^4}{5}\left(\frac{2f_B}{Lf_S}\right)^5 \end{aligned}$$

$$N_B^2 = \frac{Q^2}{12}\frac{\pi^4}{5}\left(\frac{1}{L}\right)^5$$

THIRD-ORDER DSM – MULTI-STAGE MODULATOR



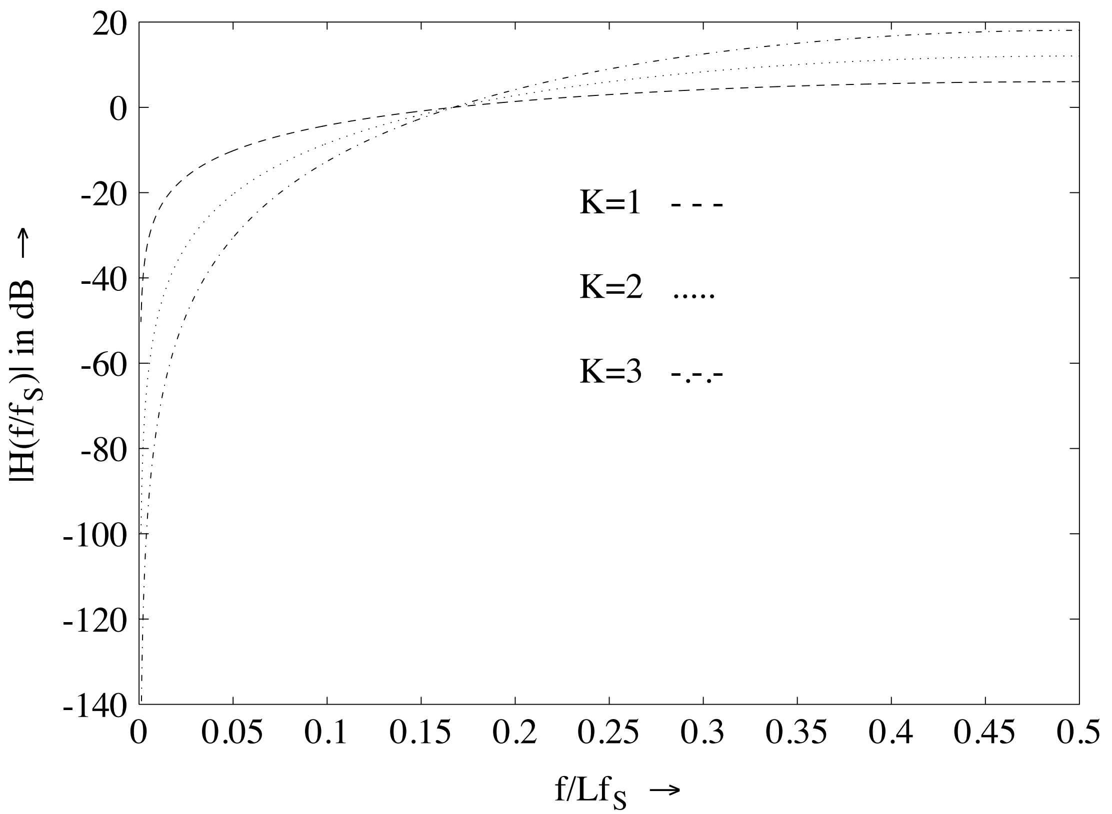
$$
\begin{aligned}
Y_1(z) &= X(z) + (1 - z^{-1})E_1(z) \\
Y_2(z) &= -E_1(z) + (1 - z^{-1})E_2(z) \\
Y_3(z) &= -E_2(z) + (1 - z^{-1})E_3(z).
\end{aligned}
$$

$$
\begin{aligned}
Y(z) &= Y_1(z) + (1 - z^{-1})Y_2(z) \\
&\quad + (1 - z^{-1})^2 Y_3(z) \\
&= X(z) + (1 - z^{-1})E_1(z) \\
&\quad - (1 - z^{-1})E_1(z) \\
&\quad + (1 - z^{-1})^2 E_2(z) \\
&\quad - (1 - z^{-1})^2 E_2(z) \\
&\quad + (1 - z^{-1})^3 E_3(z) \\
&= X(z) + \underbrace{(1 - z^{-1})^3}_{H_E(z)} E_3(z).
\end{aligned}
$$

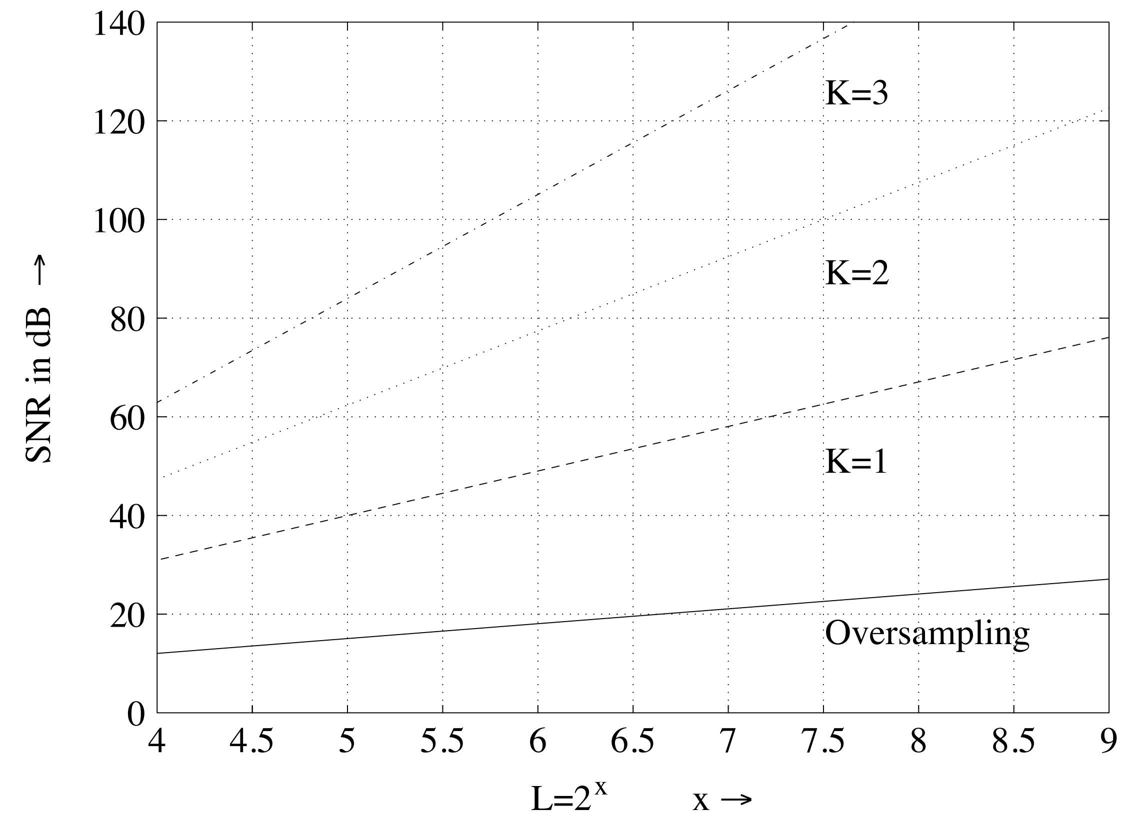
Noise power in audio band

$$N_B^2 = \frac{Q^2}{12} \frac{\pi^6}{7} \left(\frac{1}{L}\right)^7$$

SNR GAIN OF DELTA-SIGMA CONVERSION



Frequency response of error transfer functions

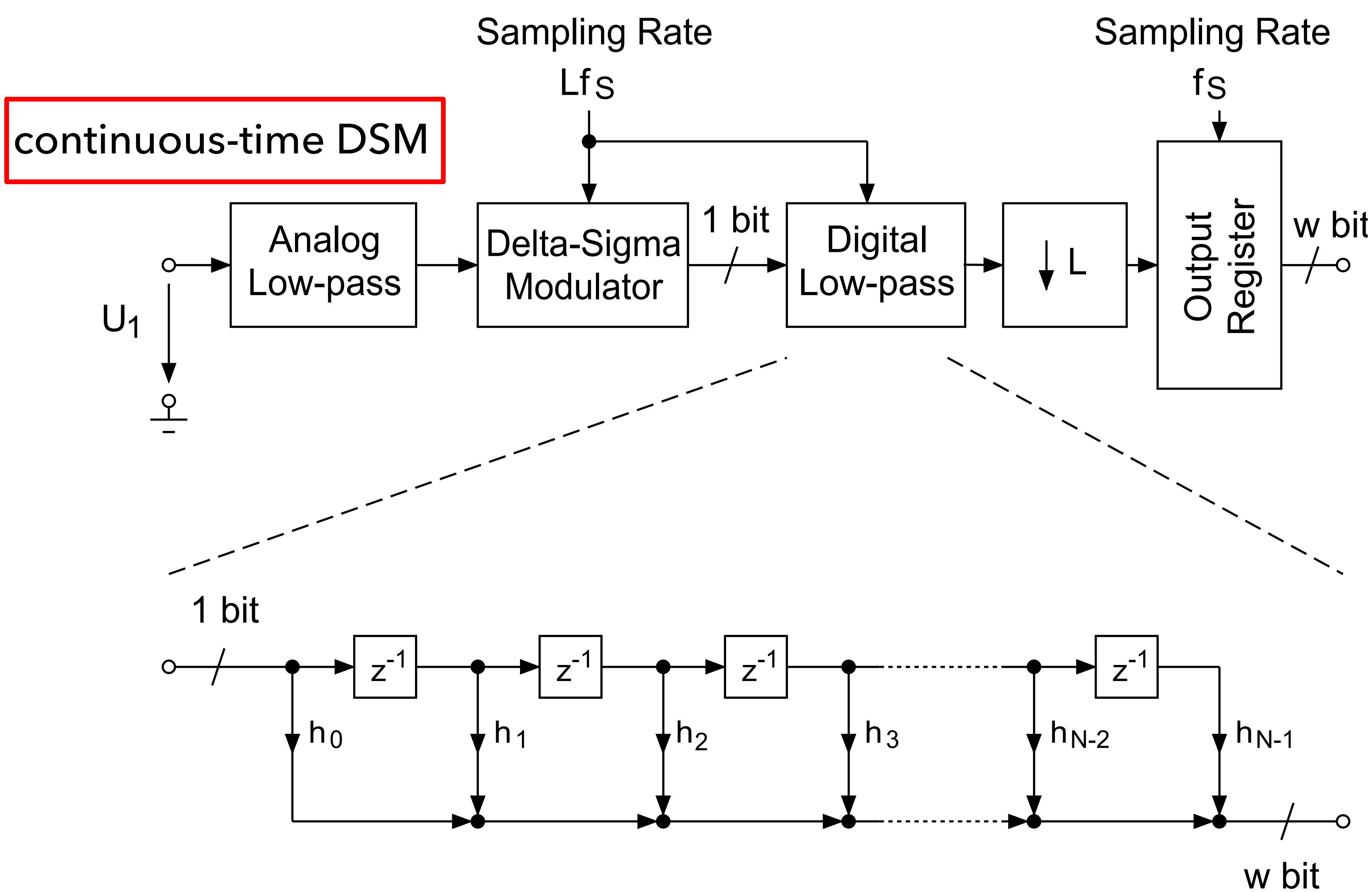


SNR improvement by oversampling and error spectrum shaping

$$H_E(z) = (1 - z^{-1})^K \text{ with } K = 1, 2, 3.$$

$$\text{SNR} = 6.02 \cdot w - 10 \log_{10}\left(\frac{\pi^{2k}}{2k+1}\right) + (2k+1) 10 \log_{10}(L) \qquad \text{dB}$$

DELTA-SIGMA AD CONVERTER



- input is 1 bit signal (0s and 1s)
- delay elements of the FIR filter are 0 and 1
- low-pass with coefficients of wordlength w bit
- sum of products (sum of coefficients) leads to w wordlength

DELTA-SIGMA DA CONVERTER