

Lecture by Udo Zölzer

- Introduction
- Quantization
- **Sampling Rate Conversion**
- AD/DA Conversion
- Equalizers
- Room Simulation
- Dynamic Range Control
- Audio Coding
- Nonlinear Processing
- Machine Learning for Audio

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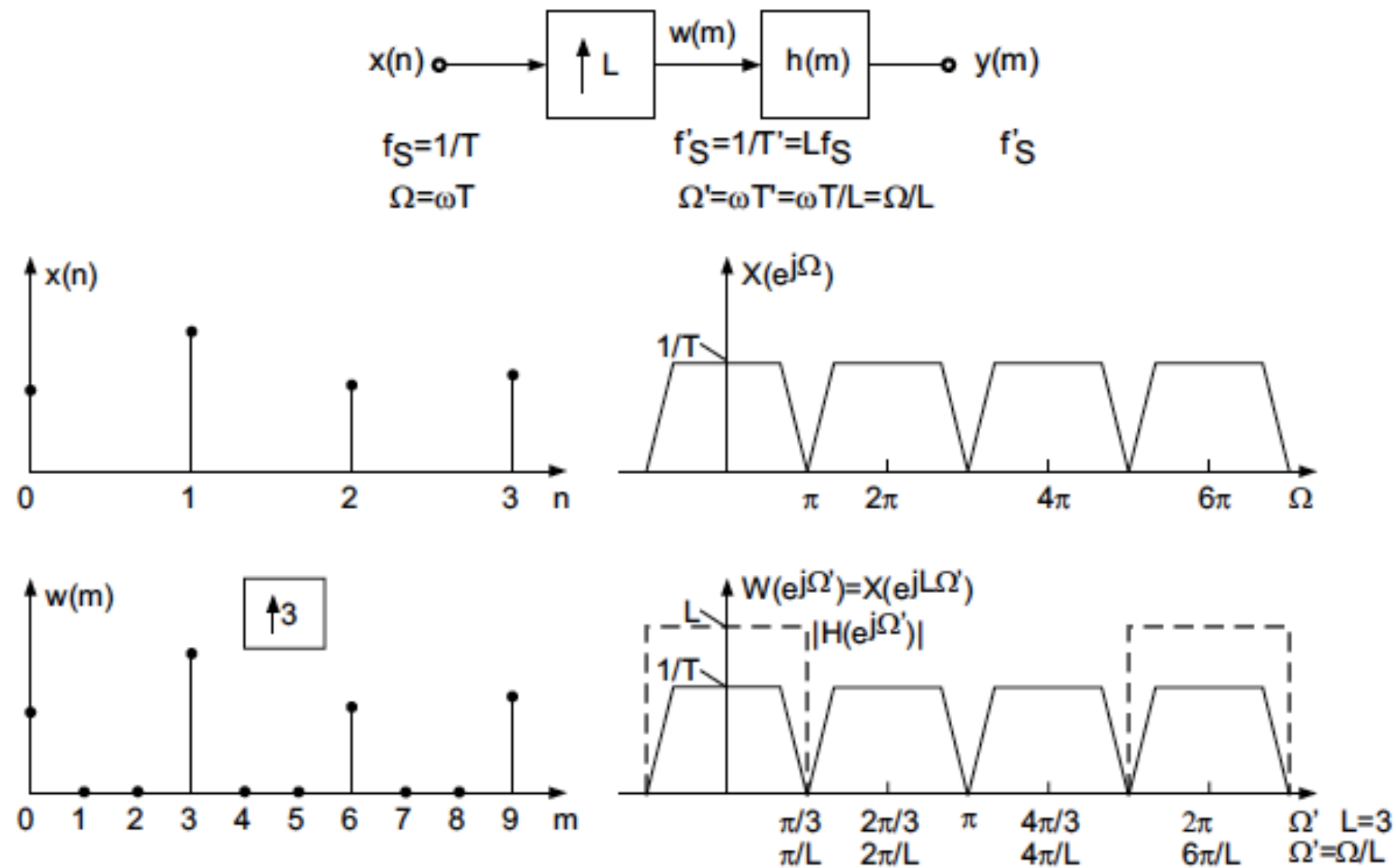
# SAMPLING RATE CONVERSION

## OUTLINE

- ▶ Basics
- ▶ Synchronous Conversion
- ▶ Asynchronous Conversion
- ▶ Interpolation Algorithms

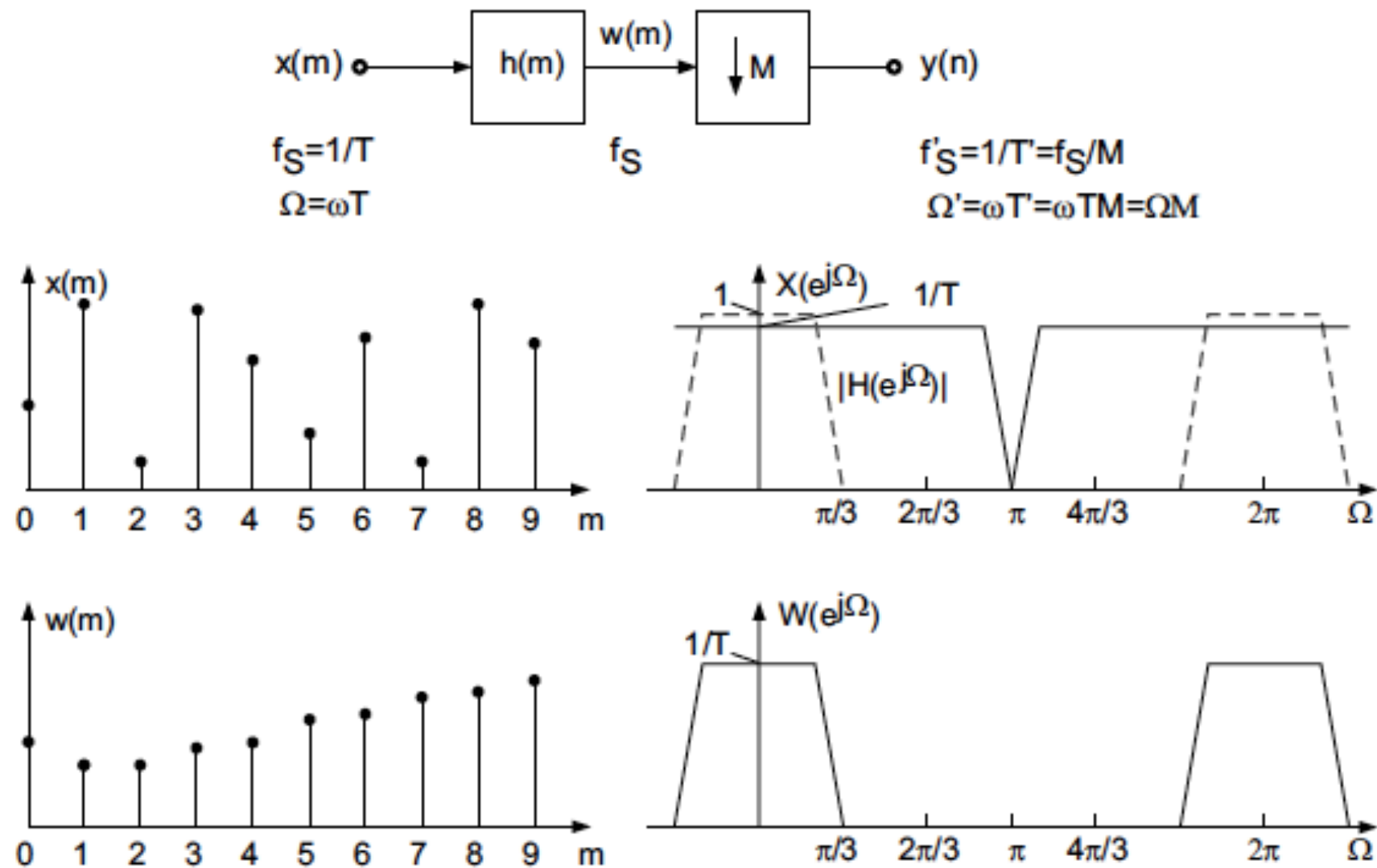
$$W(e^{j\Omega'}) = \sum_{m=-\infty}^{\infty} w(m) e^{-jm\Omega'} = \sum_{m=-\infty}^{\infty} x(m) e^{-jmL\Omega'} = X(e^{jL\Omega'})$$

# UPSAMPLING AND ANTI-IMAGE FILTER



$$Y(e^{j\Omega'}) = \frac{1}{M} \sum_{l=0}^{M-1} W(e^{j(\Omega' - 2\pi l)/M}).$$

# ANTI-ALIASING FILTER AND DOWNSAMPLING



## POLYPHASE DECOMPOSITION OF $H(z)$

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} \quad N = rM \quad \xrightarrow{\text{red arrow}} \quad H(z) = \sum_{\lambda=0}^{M-1} z^{-\lambda} H_{\lambda}(z^M)$$

$$N = 16, M = 4$$

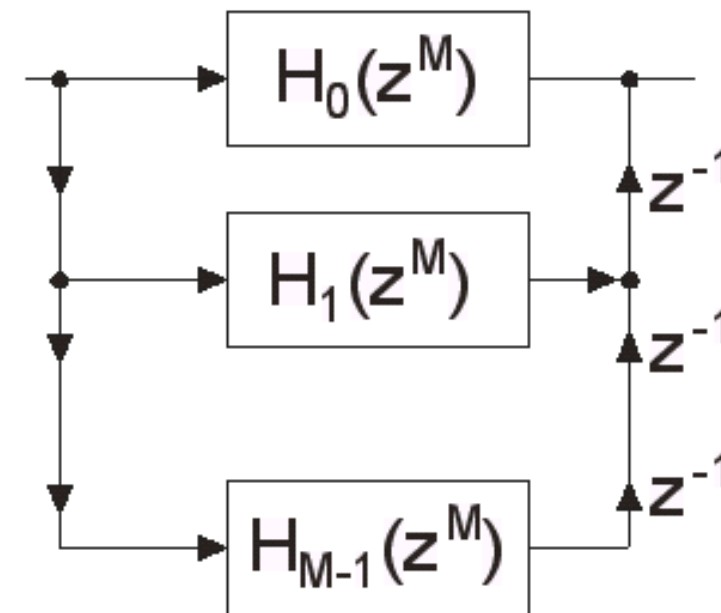
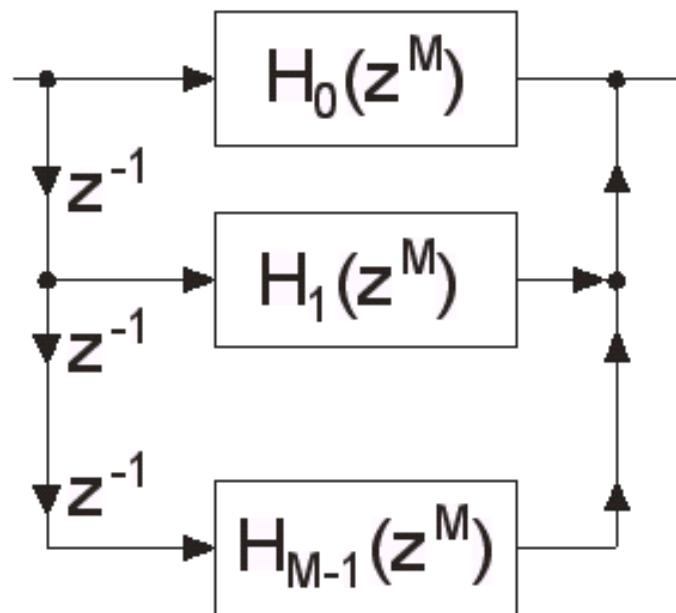
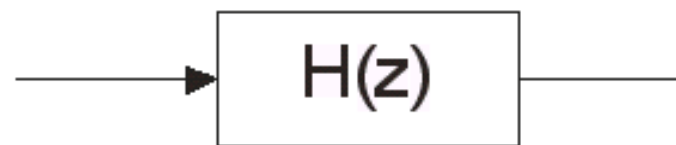
$$H(z) = \begin{array}{llll} h(0)z^0 & +h(4)z^{-4} & +h(8)z^{-8} & +h(12)z^{-12} \\ +h(1)z^{-1} & +h(5)z^{-5} & +h(9)z^{-9} & +h(13)z^{-13} \\ +h(2)z^{-2} & +h(6)z^{-6} & +h(10)z^{-10} & +h(14)z^{-14} \\ +h(3)z^{-3} & +h(7)z^{-7} & +h(11)z^{-11} & +h(15)z^{-15} \end{array}$$

$$= \begin{array}{llll} z^0[h(0) & +h(4)z^{-4} & +h(8)z^{-8} & +h(12)z^{-12}] \\ +z^{-1}[h(1) & +h(5)z^{-4} & +h(9)z^{-8} & +h(13)z^{-12}] \\ +z^{-2}[h(2) & +h(6)z^{-4} & +h(10)z^{-8} & +h(14)z^{-12}] \\ +z^{-3}[h(3) & +h(7)z^{-4} & +h(11)z^{-8} & +h(15)z^{-12}] \end{array}$$

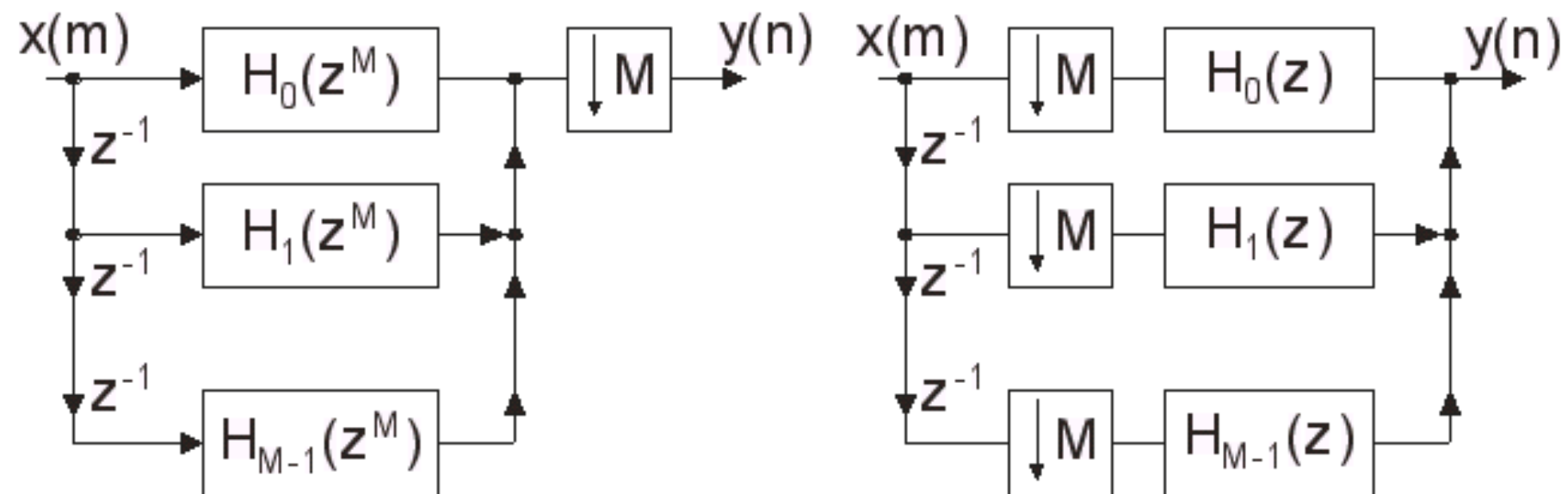
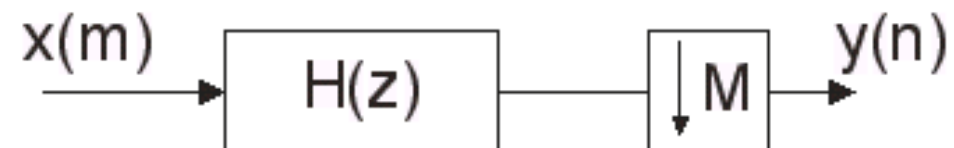
$$= H_0(z^4) + z^{-1}H_1(z^4) + z^{-2}H_2(z^4) + z^{-3}H_3(z^4)$$

# POLYPHASE FILTER STRUCTURE

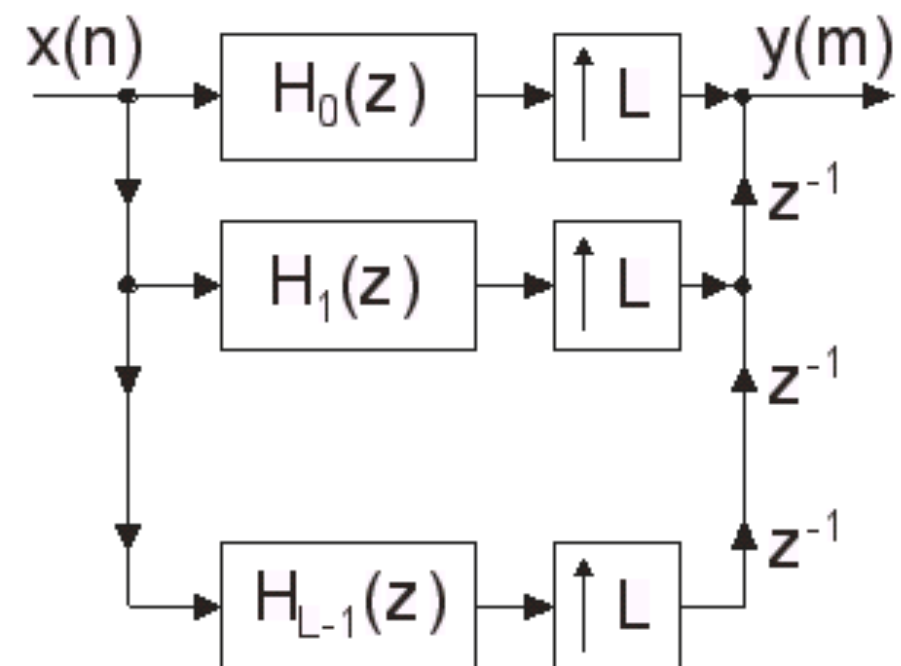
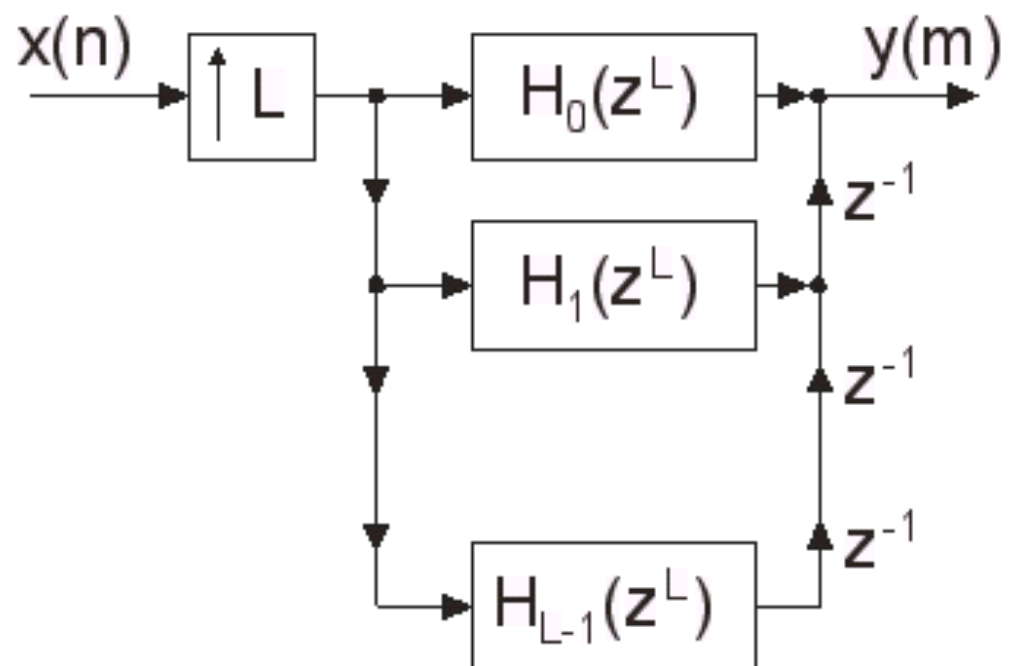
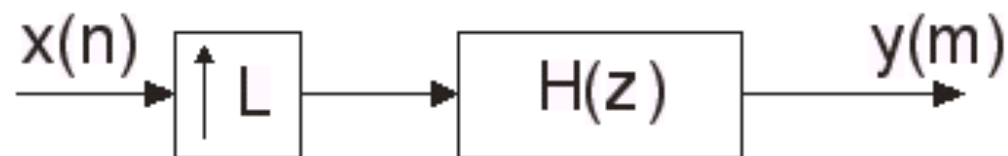
$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} \quad N = rM \quad \longrightarrow \quad H(z) = \sum_{\lambda=0}^{M-1} z^{-\lambda} H_{\lambda}(z^M)$$



# POLYPHASE FILTER AND DOWNSAMPLING

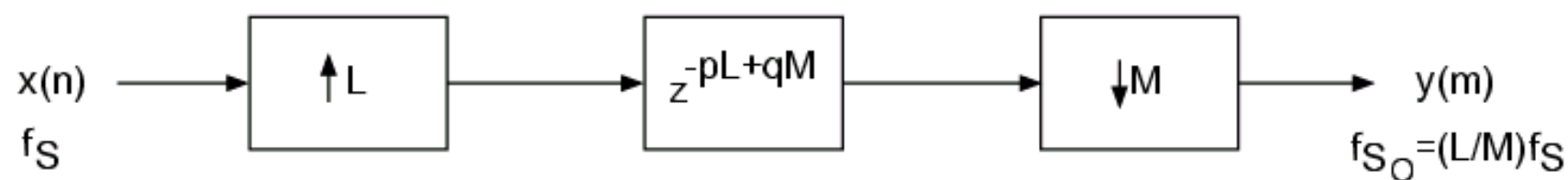
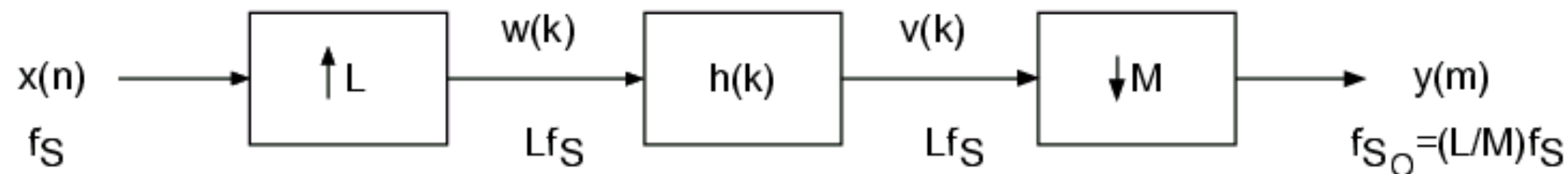


# UPSAMPLING AND POLYPHASE FILTER

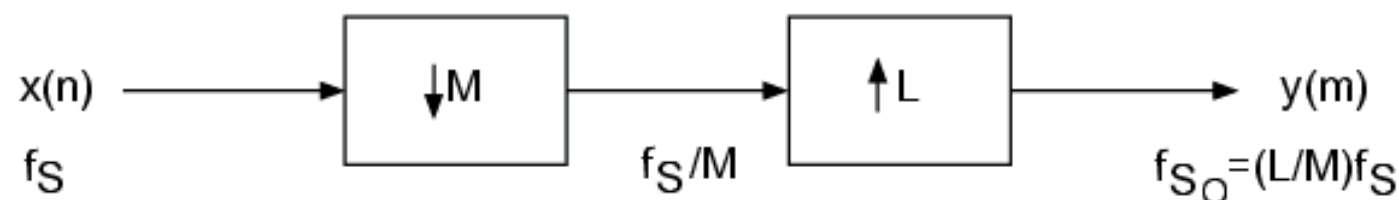
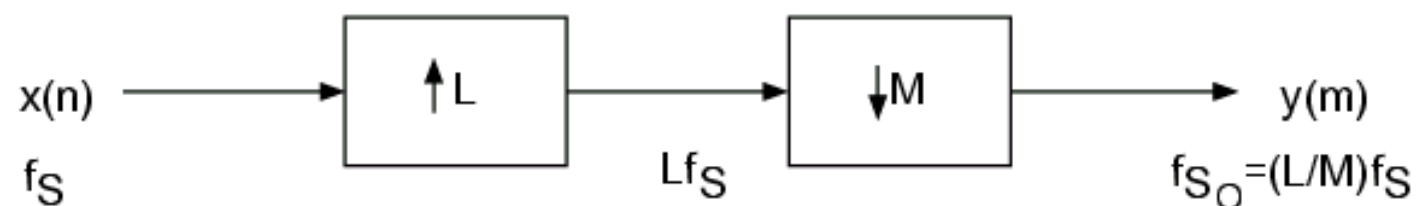
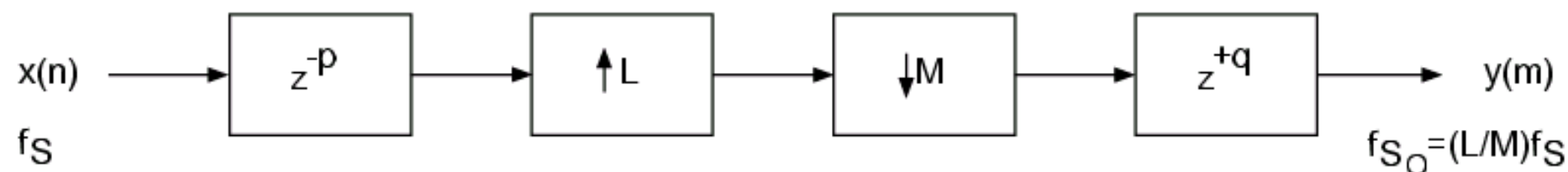




# SYNCHRONOUS CONVERSION

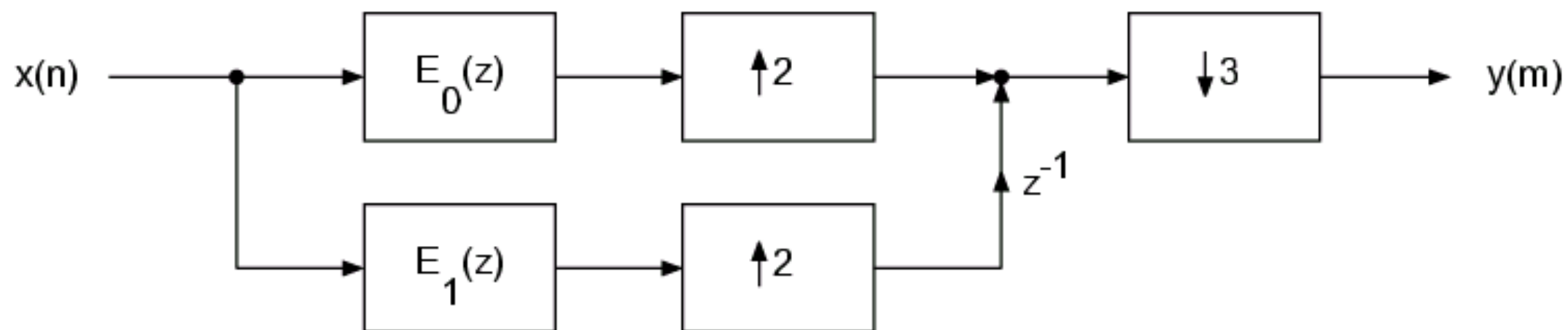
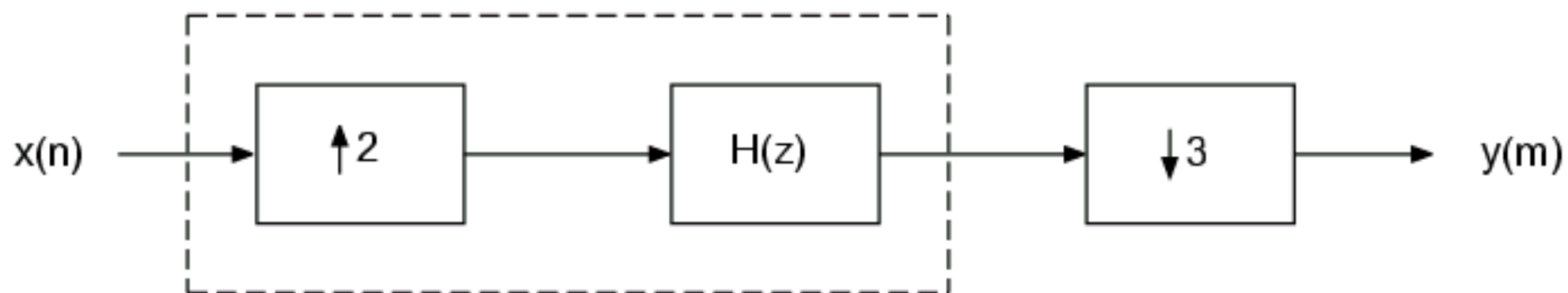


$$z^{-1} = z^{-pL} z^{qM}$$

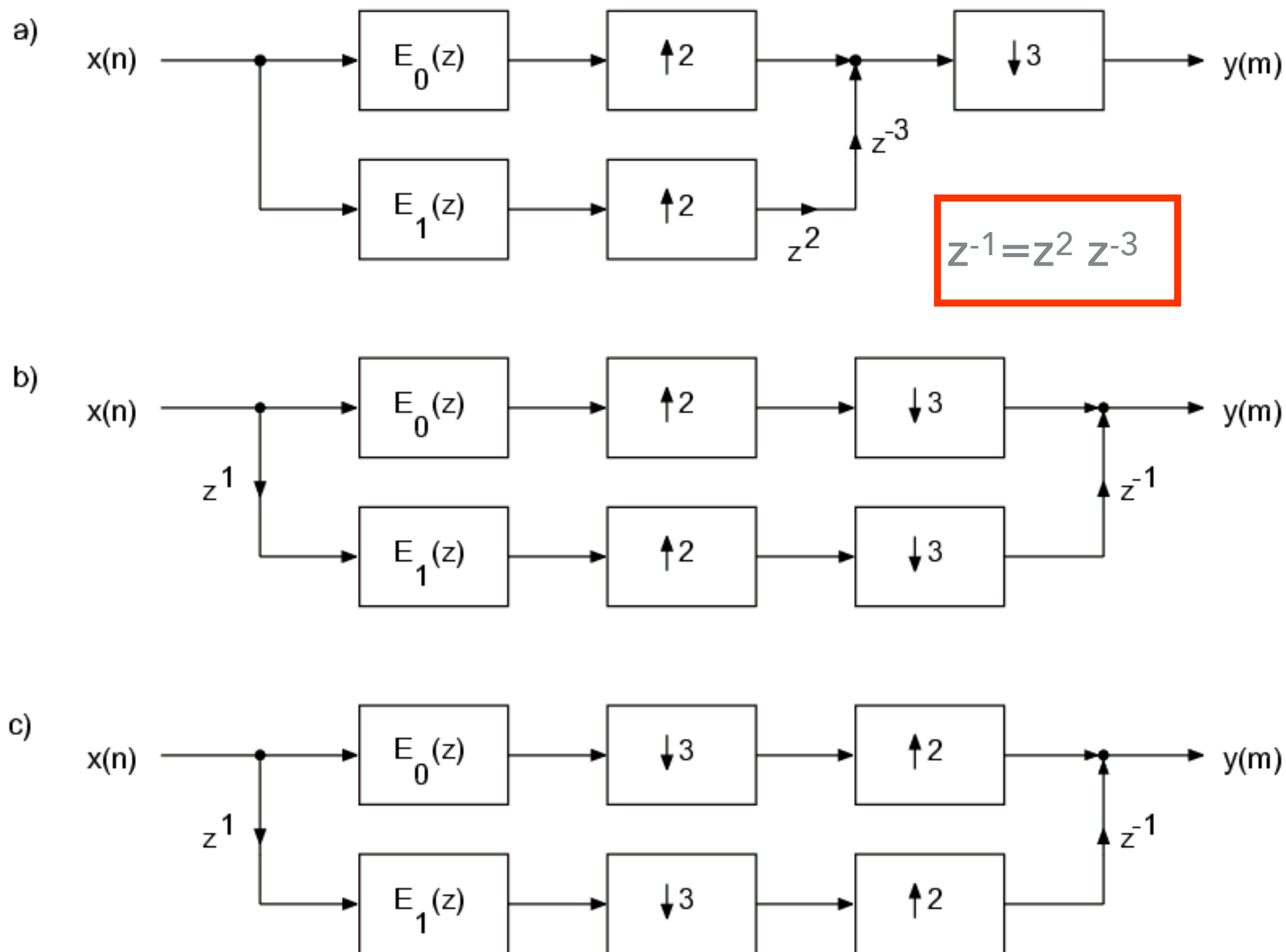


# POLYPHASE SRC

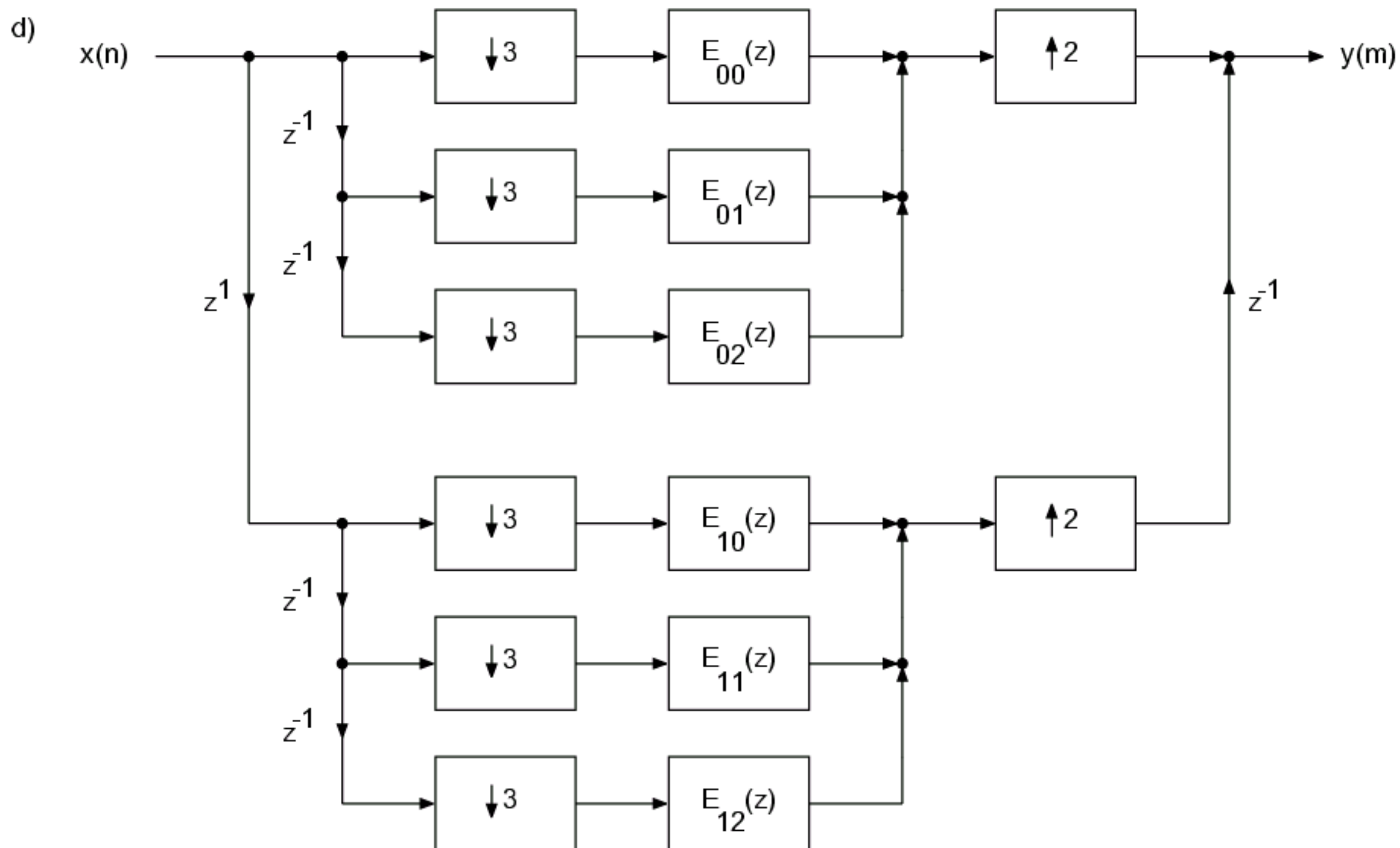
$$L/M = 2/3$$



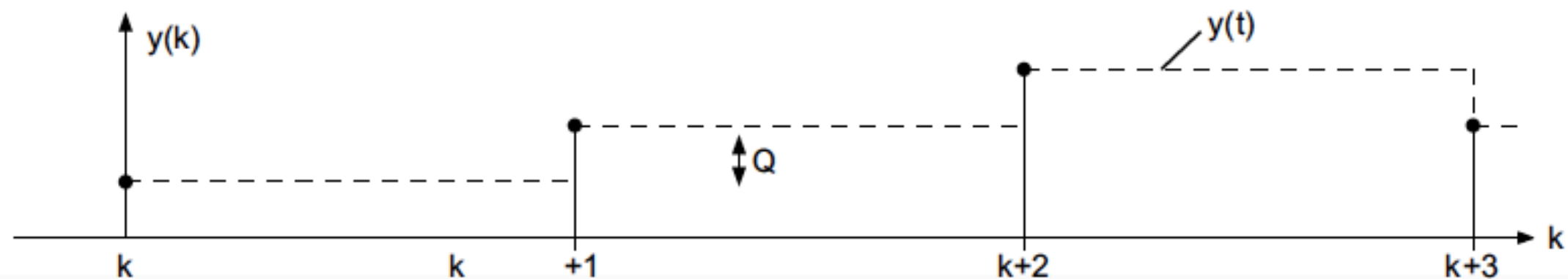
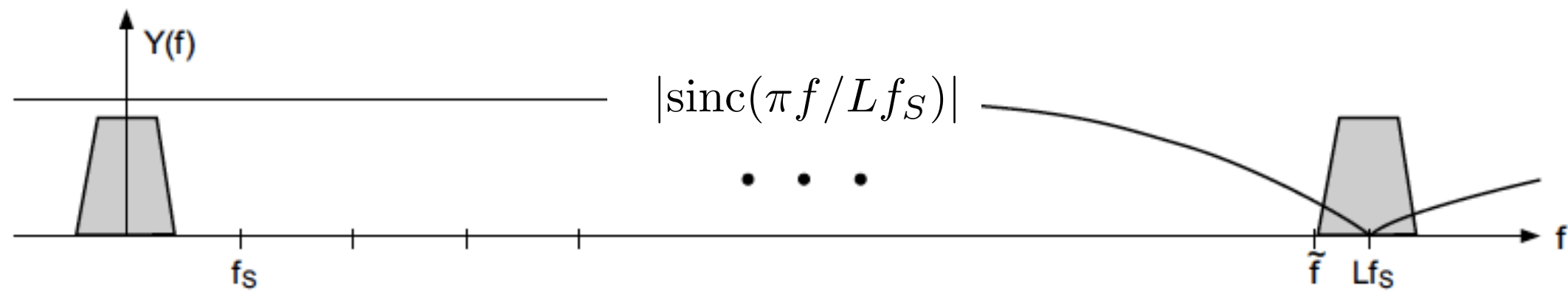
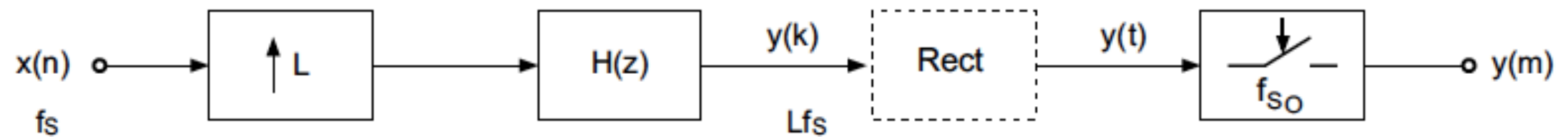
## POLYPHASE SRC - 2



## POLYPHASE SRC - 3



# ASYNCHRONOUS CONVERSION



# OVERSAMPLING FACTOR

$$\tilde{f} = (L - \frac{1}{2})f_S$$

$$\begin{aligned} E(\tilde{f}) &= \frac{\sin\left(\frac{\pi\tilde{f}}{Lf_S}\right)}{\frac{\pi\tilde{f}}{Lf_S}} \\ &= \frac{\sin\left(\frac{\pi(L-\frac{1}{2})f_S}{Lf_S}\right)}{\frac{\pi(L-\frac{1}{2})f_S}{Lf_S}} \\ &= \frac{\sin\left(\pi - \frac{\pi}{2L}\right)}{\pi - \frac{\pi}{2L}}. \end{aligned}$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

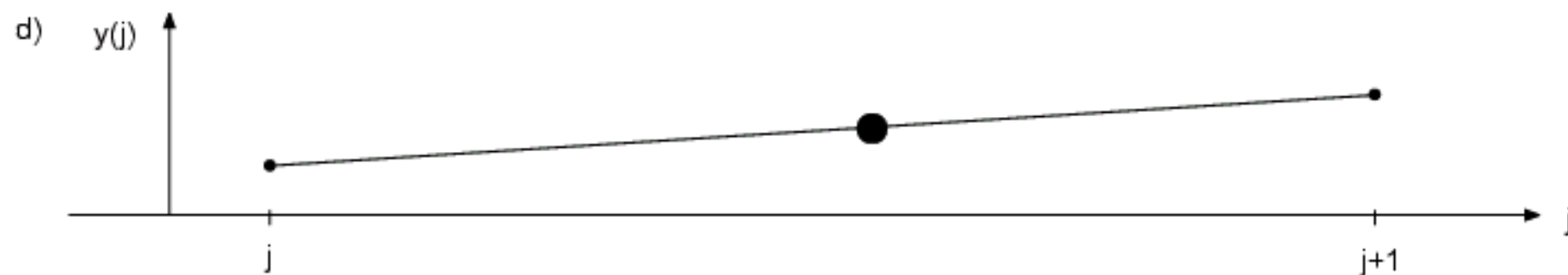
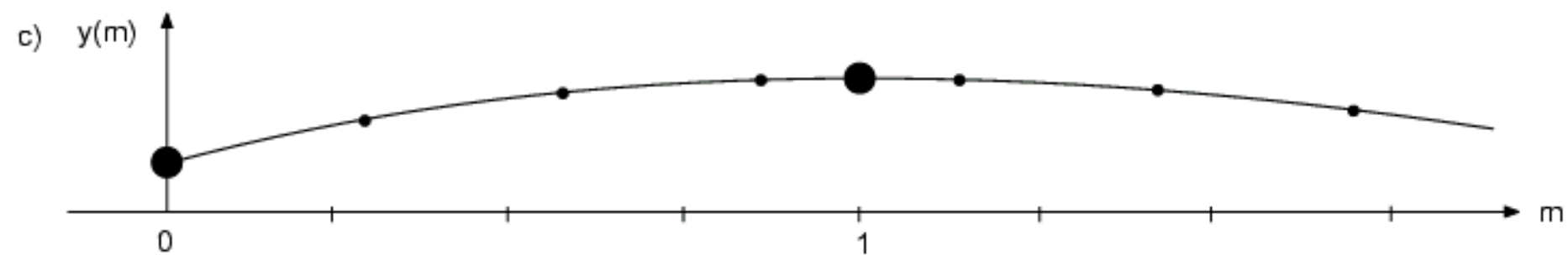
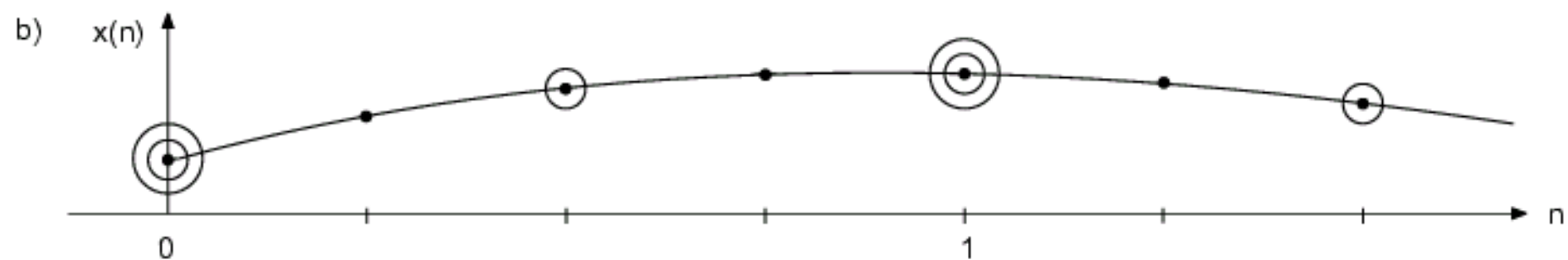
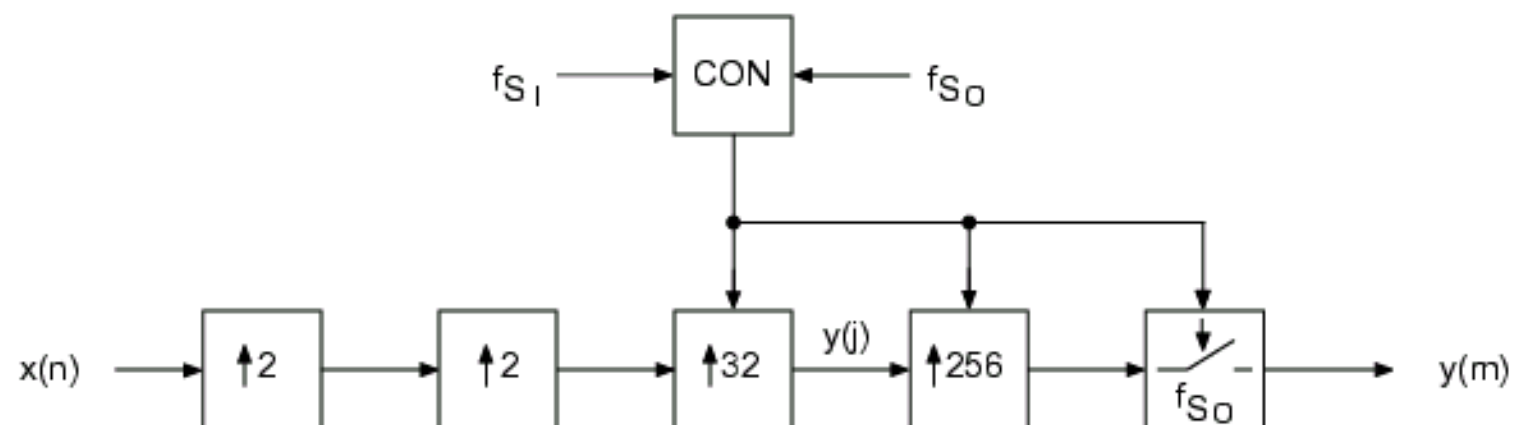
$$\begin{aligned} E(\tilde{f}) &= \frac{\sin\left(\frac{\pi}{2L}\right)}{\pi\left(1 - \frac{1}{2L}\right)} \\ &\approx \frac{\pi/2L}{\pi\left(1 - \frac{1}{2L}\right)} \\ &\approx \frac{1}{2L-1} \approx \frac{1}{2L}. \end{aligned}$$

$$\begin{aligned} \frac{Q}{2} &\geq \frac{1}{2L} \\ \frac{2^{-(w-1)}}{2} &\geq \frac{1}{2L} \end{aligned}$$

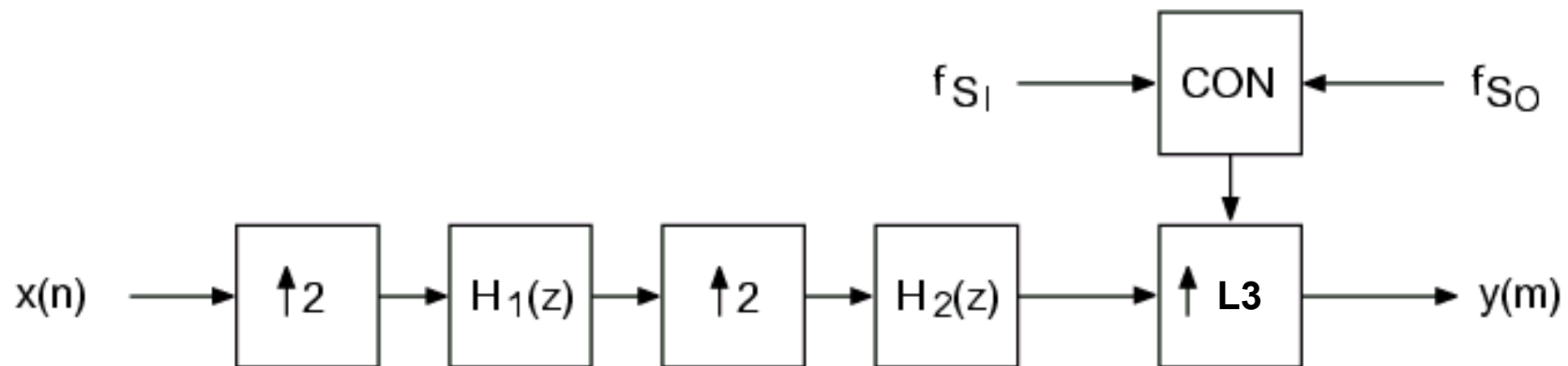
$$L \geq 2^{w-1}.$$

# MULTISTAGE SRC

a)



# MULTISTAGE SRC + INTERPOLATION ALGORITHM



- Measurement of  $f_{S_I}/f_{S_O}$  gives position  $\alpha$
- Interpolation algorithms
  - Polynomial interpolation
  - Lagrange interpolation
  - Spline interpolation

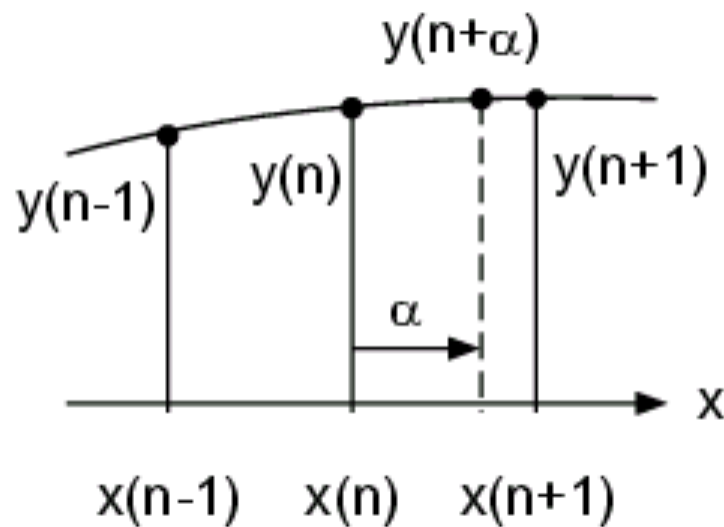
- Oversampling factors

$$L = 2^{w-1} = L_1 L_2 L_3 = 2^2 L_3$$

$$L_3 = 2^{w-3}$$



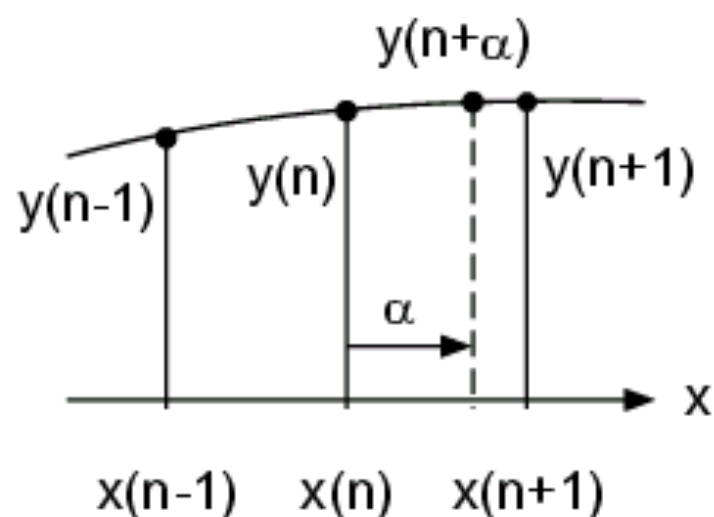
# POLYNOMIAL INTERPOLATION



$$\begin{aligned} c_{-1} &= \frac{1}{2}\alpha(\alpha - 1) \\ c_0 &= -(\alpha - 1)(\alpha + 1) = 1 - \alpha^2 \\ c_1 &= \frac{1}{2}\alpha(\alpha + 1). \end{aligned}$$

$$\begin{aligned} y(n + \alpha) &= \sum_{i=-1}^1 c_i(\alpha) y(n + i) \\ &= c_{-1}y(n - 1) + c_0y(n) + c_1y(n + 1). \end{aligned}$$

# LAGRANGE INTERPOLATION

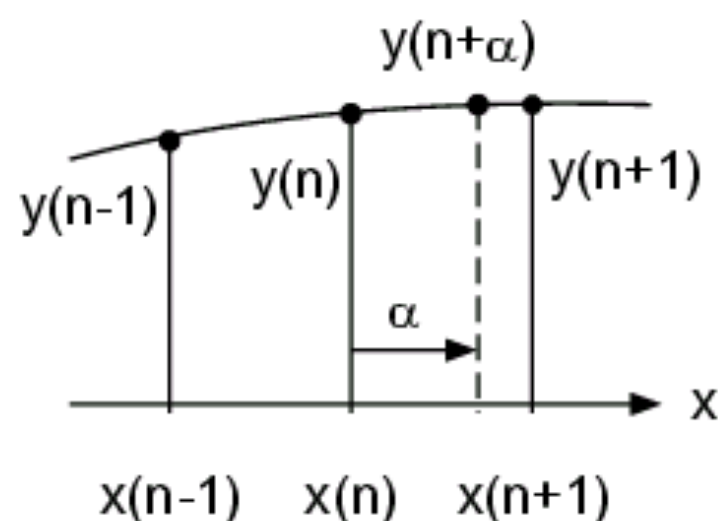


$$l_i(x(\alpha)) = \prod_{j=-\frac{N}{2}, j \neq i}^{\frac{N}{2}} \frac{\alpha - j}{i - j}$$

$$l_i(x(\alpha)) = \prod_{j=-\frac{N-1}{2}, j \neq i}^{\frac{N+1}{2}} \frac{\alpha - j}{i - j}.$$

$$y(n + \alpha) = \sum_{i=-N/2}^{N/2} l_i(\alpha) y(n + i).$$

# SPLINE INTERPOLATION



$$y(n + \alpha) = \sum_{i=-1}^1 y(n + i) N_{n-1+i}^2(\alpha)$$

$$y(n + \alpha) = \sum_{i=-1}^2 y(n + i) N_{n-2+i}^3(\alpha).$$

$$N_3^2(\alpha) = h(1) = -\frac{1}{2}\alpha^2$$

$$N_2^2(\alpha) = h(2) = -\frac{1}{2}(1 + \alpha)^2 + \frac{3}{2}\alpha^2$$

$$N_1^2(\alpha) = h(3) = -\frac{1}{2}(1 - \alpha)^2.$$

$$N_3^3(\alpha) = h(1) = \frac{1}{6}\alpha^3$$

$$N_2^3(\alpha) = h(2) = \frac{1}{6}(1 + \alpha)^3 - \frac{2}{3}\alpha^3$$

$$N_1^3(\alpha) = h(3) = \frac{1}{6}(2 - \alpha)^3 - \frac{2}{3}(1 - \alpha)^3$$

$$N_0^3(\alpha) = h(4) = \frac{1}{6}(1 - \alpha)^3.$$



## Sampling Rate Conversion

## Upsampling

$$x(n) \xrightarrow{\uparrow L} w(m) = \begin{cases} x\left(\frac{m}{L}\right) & m=0, \pm L, \dots \\ 0 & \text{else} \end{cases}$$

$$f_s = \frac{1}{T}$$

$$\Omega = \omega T$$

$$f_s' = L f_s = \frac{1}{T'}$$

$$T' = \frac{T}{L}$$

$$\Omega' = \frac{\omega T}{L} = \frac{\Omega}{L}$$

$$W(z) = \sum_{m=-\infty}^{+\infty} w(m) \cdot z^{-m}$$

$$= \sum_{m=-\infty}^{+\infty} x\left(\frac{m}{L}\right) \cdot z^{-mL}$$

$$= X(z^L)$$

$$W(e^{j\Omega'}) = X(e^{j\Omega' L})$$

## Downsampling

$$w(m) \xrightarrow{\downarrow M} y(n)$$

$$f_s = \frac{1}{T}$$

$$\Omega = \omega T$$

$$f_s' = \frac{f_s}{M} = \frac{1}{T'}$$

$$T' = M \cdot T$$

$$\Omega' = \Omega M$$

$$w'(m) = \begin{cases} w(m) & m=0, \pm M, \dots \\ 0 & \text{else} \end{cases}$$

$$= w(m) \cdot \frac{1}{M} \sum_{l=0}^{M-1} e^{j2\pi l m / M}$$

$$y(n) = w'(M \cdot n) = w(Mn)$$

$$Y(z) = \sum_{n=-\infty}^{+\infty} y(n) \cdot z^{-n} = \sum_{n=-\infty}^{+\infty} w'(Mn) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} w'(n) \cdot z^{-n/M}$$

$$Y(e^{j\Omega'}) = \frac{1}{M} \sum_{l=0}^{M-1} W(e^{j(\Omega' - 2\pi l)/M})$$



**Derivation for downsampling**

$$w'(m) = w(m) \cdot \frac{1}{M} \sum_{l=0}^{M-1} e^{j2\pi lm/M}$$

$$y(n) = w'(Mm) = w(Mm)$$

$$Y(z) = \sum_n y(n) z^{-n} = \sum_n w'(Mn) z^{-n}; Mn = k \rightarrow n = \frac{k}{M} \rightarrow k = n$$

$$= \sum_n w'(n) z^{-n/M}$$

$$= \sum_n w(n) \frac{1}{M} \sum_{l=0}^{M-1} e^{j2\pi ln/M} z^{-n/M}$$

$$= \frac{1}{M} \sum_{l=0}^{M-1} \left[ \sum_n w(n) \underbrace{e^{j2\pi ln/M} z^{-n/M}}_{(e^{-j2\pi l/M} z^{1/M})^{-n}} \right]$$

$$= \frac{1}{M} \sum_{l=0}^{M-1} W(e^{-j2\pi l/M} z^{1/M})$$

$$Y(e^{j\Omega'}) = \frac{1}{M} \sum_{l=0}^{M-1} W(e^{-j2\pi l/M} e^{j\Omega'/M}) = \frac{1}{M} \sum_{l=0}^{M-1} W(e^{j(\Omega' - 2\pi l)/M})$$